Further trigonometry - Synthesis test 5

Name: _____ Date: _____

10 questions | 49 marks | 75 minutes

A more demanding paper combining reasoning, error analysis, exact values and model construction. Explain exclusions and check the meaning of every result.

A scientific calculator may be used.

1. A student calculates the length of an arc of radius 5 cm and angle 30° as 150 cm. [4 marks]

(a) Find the correct length and explain the student error. [2 marks]

(b) Find $\sin(-\frac{5\pi}{6})$ and $\cos(-\frac{5\pi}{6})$ exactly. [2 marks]

2. A graph has minimum -2 at $x=1$, maximum 6 at $x=5$, and the next minimum at $x=9$. [5 marks]

(a) Construct a cosine rule that matches these observations. [3 marks]

(b) Find the two midline crossings between the consecutive minima. [2 marks]

3. Solve $\cos 2x = \cos x$ on $[0, 2\pi)$. [5 marks]

(a) Find every solution algebraically. [3 marks]

(b) Explain why dividing first by $\cos x$ is an unnecessarily risky method, even though its zeros are not solutions here. [2 marks]

4. An angle A is in quadrant II and $\sin A = \frac{3}{5}$. [5 marks]

(a) Find $\sin(A + \frac{\pi}{6})$ and $\cos(A + \frac{\pi}{6})$ exactly. [3 marks]

(b) Identify the quadrant of $A + \frac{\pi}{6}$ using those signs. [2 marks]

5. An angle θ lies in quadrant IV with $\sin\theta = -\frac{4}{5}$ and $0 \leq \theta < 2\pi$. [5 marks]

(a) Find $\cos(\frac{\theta}{2})$ exactly. [3 marks]

(b) Find $\tan(\frac{\theta}{2})$ exactly. [2 marks]

6. A combined signal is $f(x) = a\sin x + b\cos x$, with $a > 0$, amplitude 10 and $f(0) = -6$. [5 marks]

(a) Find a, b and express the signal as one shifted sine. [3 marks]

(b) Find the smallest positive zero of the signal. [2 marks]

7. Find intersections of $y = \sec x$ and $y = \csc x$ on $[0, 2\pi)$. [5 marks]

(a) Find the input values, stating the domain restrictions used. [3 marks]

(b) Give the two intersection coordinates. [2 marks]

8. A student writes $\sqrt{1 - \sin^2 x} = \cos x$ for every real x. [4 marks]

(a) Correct the statement and explain it. [2 marks]

(b) Give a counterexample to the student statement using an exact angle. [2 marks]

9. An ideal tide repeats every 8 hours. Its height is 1 m at $t = 0$ and 5 m at $t = 4$. Assume these are a minimum and the next maximum. [6 marks]

(a) Construct a model and calculate its height after 2 hours. [3 marks]

(b) Find the duration for which the tide is at least 4 m in one 8-hour cycle. [3 marks]

10. Define $g(x) = 1 + 2\sin x \cos x$. [5 marks]

(a) Find its exact range and period. [2 marks]

(b) Solve $g(x) = \frac{1}{2}$ on $[0, 2\pi)$. [3 marks]