Further trigonometry - Practice test 4

Name: _____ Date: _____

10 questions | 43 marks | 65 minutes

Use exact calculation and periodic models in unfamiliar but fully specified situations. Interpret all units and restrictions.

A scientific calculator may be used.

1. A circular track has radius 12 m. A runner moves through an angle of $\frac{7\pi}{12}$. [4 marks]

(a) Find the distance travelled along the track. [2 marks]

(b) Find the area of the corresponding sector. [2 marks]

2. A graph is modelled by $p(x) = -2 + 4\cos(3x + \pi)$. [4 marks]

(a) Find its period, horizontal shift and range. [2 marks]

(b) Give an equivalent rule with no phase shift. [2 marks]

3. Solve $\tan(x-\frac{\pi}{6})=1$ on $0\le x<2\pi$. [4 marks]

(a) Give every solution. [2 marks]

(b) State the family of excluded inputs of the original tangent expression. [2 marks]

4. Acute angles A, B satisfy $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$. [4 marks]

(a) Find $\tan(A+B)$ exactly. [2 marks]

(b) Find $A+B$ exactly, justifying the selected angle. [2 marks]

5. Work with double-angle identities. [4 marks]

(a) Show that $4\sin^2 x\cos^2 x = \sin^2 2x$. [2 marks]

(b) Find every x in $[0, \pi)$ for which $4\sin^2 x\cos^2 x = 1$. [2 marks]

6. Two same-frequency signals combine as $S(x) = 9\cos x + 12\sin x$. [4 marks]

(a) Find the largest possible value of S and express S as one cosine. [2 marks]

(b) Find the first positive input giving this largest value. [2 marks]

7. A student proposes that $\sec x$ has a zero at $x = \frac{\pi}{2}$. [4 marks]

(a) Explain the error using the definition. [2 marks]

(b) Solve $2\cot x = \sqrt{3}$ on $[0, \pi)$. [2 marks]

8. Prove $\frac{\cos x}{1 + \sin x} = \frac{1 - \sin x}{\cos x}$ on the common domain. [5 marks]

(a) Give a valid one-sided proof. [3 marks]

(b) Give a point where the original left expression exists but the right expression does not. [2 marks]

9. An ideal observation wheel has axle height 15 m, radius 10 m and period 60 s. A seat starts at its highest point. [5 marks]

(a) Write a height model in metres. [2 marks]

(b) For how long in one turn is the seat at least 20 m above ground? [3 marks]

10. A sinusoid has form $f(x) = D + A\sin x + B\cos x$, with $f(0) = 5$, $f(\pi) = 1$ and $f(\frac{\pi}{2}) = 7$. [5 marks]

(a) Find D, A, B. [3 marks]

(b) Find its exact range. [2 marks]