Further trigonometry - Practice test 4

10 questions | 43 marks | 65 minutes

Use exact calculation and periodic models in unfamiliar but fully specified situations. Interpret all units and restrictions.

Equivalent correct methods earn the corresponding marks.

1. A circular track has radius 12 m. A runner moves through an angle of $\frac{7\pi}{12}$. [4 marks]

(a) Find the distance travelled along the track. [2 marks]

Use $s = r\theta = 12(\frac{7\pi}{12})$.

Answer: 7π m.

(b) Find the area of the corresponding sector. [2 marks]

Use $\frac{1}{2}(12^2)(\frac{7\pi}{12}) = 42\pi$.

Answer: 42π m^2.

2. A graph is modelled by $p(x) = -2 + 4\cos(3x + \pi)$. [4 marks]

(a) Find its period, horizontal shift and range. [2 marks]

Factor $3x + \pi = 3(x + \frac{\pi}{3})$; midline -2 and amplitude 4.

Answer: Period $\frac{2\pi}{3}$; left shift $\frac{\pi}{3}$; range $[-6, 2]$.

(b) Give an equivalent rule with no phase shift. [2 marks]

Use $\cos(u + \pi) = -\cos u$.

Answer: $p(x) = -2 - 4\cos 3x$.

3. Solve $\tan(x-\frac{\pi}{6})=1$ on $0\le x<2\pi$. [4 marks]

(a) Give every solution. [2 marks]

The argument equals $\frac{\pi}{4}+k\pi$; add $\frac{\pi}{6}$ and select two values.

Answer: $x=\frac{5\pi}{12},\frac{17\pi}{12}$.

(b) State the family of excluded inputs of the original tangent expression. [2 marks]

The argument cannot equal $\frac{\pi}{2}+k\pi$.

Answer: $x=\frac{2\pi}{3}+k\pi$.

4. Acute angles A,B satisfy $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$. [4 marks]

(a) Find $\tan(A+B)$ exactly. [2 marks]

$\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}=\frac{\frac{5}{6}}{\frac{5}{6}}=1$.

Answer: 1.

(b) Find $A+B$ exactly, justifying the selected angle. [2 marks]

The sum of two acute angles lies in $(0,\pi)$; tangent 1 there occurs only at $\frac{\pi}{4}$.

Answer: $\frac{\pi}{4}$.

5. Work with double-angle identities. [4 marks]

(a) Show that $4\sin^2 x\cos^2 x=\sin^2 2x$. [2 marks]

Both sides are squares, so the result holds for every real x.

Answer: Square $\sin 2x=2\sin x\cos x$.

(b) Find every x in $[0,\pi)$ for which $4\sin^2 x\cos^2 x=1$. [2 marks]

The identity gives $\sin^2 2x=1$, so $2x=\frac{\pi}{2},\frac{3\pi}{2}$ in $[0,2\pi)$.

Answer: $x=\frac{\pi}{4},\frac{3\pi}{4}$.

6. Two same-frequency signals combine as $S(x) = 9\cos x + 12\sin x$. [4 marks]

(a) Find the largest possible value of S and express S as one cosine. [2 marks]

Match cosine coefficient $15\cos\phi = 9$ and sine coefficient $15\sin\phi = 12$.

Answer: 15; $S = 15\cos(x - \phi)$ with $\phi = \tan^{-1}(\frac{4}{3})$.

(b) Find the first positive input giving this largest value. [2 marks]

The cosine argument is zero at $x = \phi$; ϕ is acute.

Answer: $x = \tan^{-1}(\frac{4}{3})$.

7. A student proposes that $\sec x$ has a zero at $x = \frac{\pi}{2}$. [4 marks]

(a) Explain the error using the definition. [2 marks]

Its denominator cosine is zero; the numerator 1 never vanishes.

Answer: Secant is undefined there, not zero.

(b) Solve $2\cot x = \sqrt{3}$ on $[0, \pi)$. [2 marks]

Cotangent $\frac{\sqrt{3}}{2}$ gives tangent $\frac{2}{\sqrt{3}}$.

The solution is in quadrant I, and tangent has period π.

Answer: $x = \tan^{-1}(\frac{2}{\sqrt{3}}) \approx 0.857$ rad.

8. Prove $\frac{\cos x}{1 + \sin x} = \frac{1 - \sin x}{\cos x}$ on the common domain. [5 marks]

(a) Give a valid one-sided proof. [3 marks]

For $\cos x \neq 0$, multiply numerator/denominator by $1 - \sin x$.

The denominator becomes $1 - \sin^2 x = \cos^2 x$; cancel one cosine.

Answer: Multiply the left side by the conjugate and simplify.

(b) Give a point where the original left expression exists but the right expression does not. [2 marks]

At $\frac{\pi}{2}$, the left is $\frac{0}{1+1} = 0$, while the right is $\frac{0}{0}$ and undefined.

Answer: $x = \frac{\pi}{2}$.

9. An ideal observation wheel has axle height 15 m, radius 10 m and period 60 s. A seat starts at its highest point. [5 marks]

(a) Write a height model in metres. [2 marks]

The midline 15, amplitude 10 and frequency $\frac{2\pi}{60}$ match the context.

Answer: $h(t) = 15 + 10\cos(\frac{\pi t}{30})$.

(b) For how long in one turn is the seat at least 20 m above ground? [3 marks]

Cosine must be at least $\frac{1}{2}$, giving $0 \le t \le 10$ and $50 \le t \le 60$.

The interval lengths add to 20.

Answer: 20 seconds in total.

10. A sinusoid has form $f(x) = D + A\sin x + B\cos x$, with $f(0) = 5$, $f(\pi) = 1$ and $f(\frac{\pi}{2}) = 7$. [5 marks]

(a) Find D, A, B. [3 marks]

The first two observations give $D + B = 5$, $D - B = 1$, so $D = 3$, $B = 2$.

The third gives $D + A = 7$.

Answer: $D = 3$, $A = 4$, $B = 2$.

(b) Find its exact range. [2 marks]

Resultant amplitude is $\sqrt{4^2 + 2^2} = 2\sqrt{5}$.

Answer: $[3 - 2\sqrt{5}, 3 + 2\sqrt{5}]$.