Further trigonometry - Practice test 3

Name: _____ Date: _____

10 questions | 42 marks | 65 minutes

Connect exact identities, restricted domains and graph information. Justify each branch of a solution set.

A scientific calculator may be used.

1. Let θ have reference angle $\frac{\pi}{3}$ in quadrant II. [4 marks]

(a) Find $\sec\theta$ and $\cot\theta$ exactly. [2 marks]

(b) Find $\sin(\theta+\pi)$ exactly. [2 marks]

2. A sinusoidal signal has maximum 9 and minimum -3. It crosses its midline upwards at $x=1$, and its next upward midline crossing is at $x=5$. [4 marks]

(a) Construct a sine rule. [2 marks]

(b) Find its first maximum for $x\geq 1$. [2 marks]

3. Solve $2\sin^2 x + \sin x - 1 = 0$ on $[0, 2\pi)$. [5 marks]

(a) Factor the equation and find every angle. [3 marks]

(b) Explain why neither algebraic root is rejected before finding angles. [2 marks]

4. Use $105^\circ = 60^\circ + 45^\circ$. [4 marks]

(a) Find $\cos 105^\circ$ exactly. [2 marks]

(b) Find $\tan 105^\circ$ exactly. [2 marks]

5. An angle x is acute and $\cos 2x = \frac{7}{25}$. [4 marks]

(a) Find $\sin x$ and $\cos x$ exactly. [2 marks]

(b) Find $\sin 4x$. [2 marks]

6. Consider $w(x) = 7\sin x - 24\cos x$. [4 marks]

(a) Find its amplitude and express it as $R\sin(x+\phi)$ with $-\frac{\pi}{2} < \phi < 0$. [2 marks]

(b) Find its maximum and the first nonnegative input where it occurs. [2 marks]

7. Consider $r(x) = 3\csc(2x) - 1$. [4 marks]

(a) State the period and domain. [2 marks]

(b) State its range. [2 marks]

8. A student simplifies $\frac{\sin 2x}{1 - \cos 2x}$ to $\cot x$. [5 marks]

(a) Show the simplification with restrictions. [3 marks]

(b) Does the original expression equal zero at $x = \frac{\pi}{2}$? Explain. [2 marks]

9. A rotating beacon has model brightness $B(t) = 4 + 3\sin(\frac{\pi t}{5})$, in arbitrary units, for $0 \le t \le 10$ seconds. [4 marks]

(a) Find when brightness equals $\frac{11}{2}$. [2 marks]

(b) Find the duration for which brightness exceeds $\frac{11}{2}$. [2 marks]

10. An expression $a\sin x + b\cos x$ has amplitude 5 and is equal to 3 at $x = 0$. [4 marks]

(a) Find the possible pairs (a, b). [2 marks]

(b) What extra observation would distinguish the two possibilities without differentiation? [2 marks]