Further trigonometry - Practice test 2

Name: _____ Date: _____

10 questions | 40 marks | 65 minutes

Use representations, exact reasoning and graph restrictions. Supply exact answers where possible and state domains.

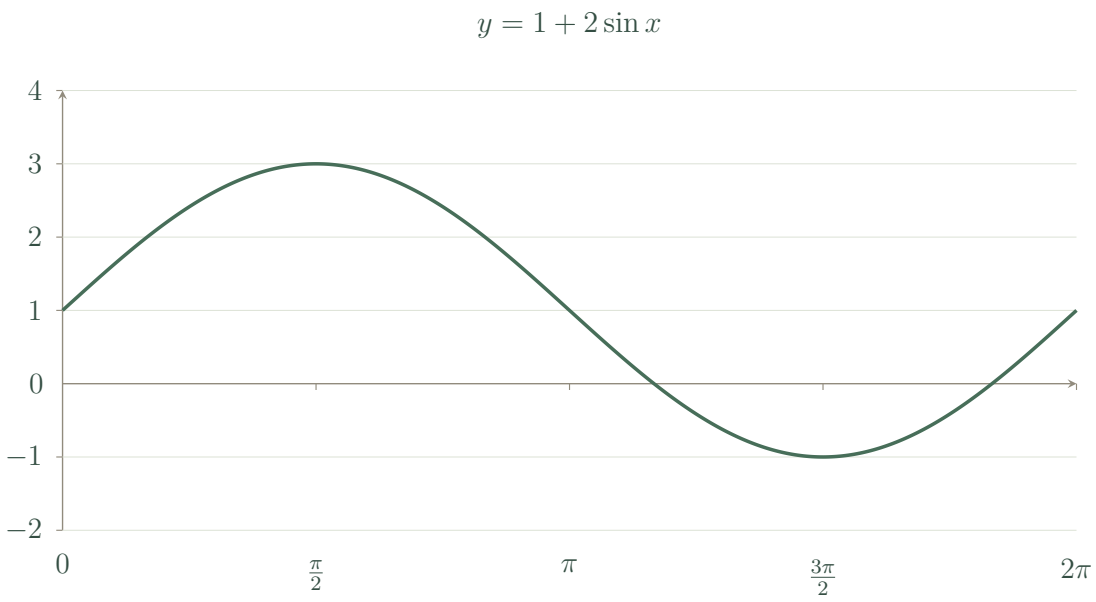
A scientific calculator may be used.

1. A point on the unit circle lies in quadrant III and has horizontal coordinate $-\frac{12}{13}$. [4 marks]

(a) Find its vertical coordinate. [2 marks]

(b) Find $\tan\theta$ and $\cos(2\theta)$. [2 marks]

2. The graph below shows $y = 1 + 2\sin x$. [4 marks]



(a) State the midline and range. [2 marks]

(b) Give the coordinates of both turning points on $[0, 2\pi]$. [2 marks]

3. Solve equations on the interval $0 \leq x \leq 2\pi$. [4 marks]

(a) Solve $\cos(\frac{x}{2}) = 0$. [2 marks]

(b) Solve $\sin^2 x = \frac{1}{4}$. [2 marks]

4. Develop exact angle values. [4 marks]

(a) Find $\tan 15^\circ$ exactly. [2 marks]

(b) Use the sum formula to show $\sin(\pi + x) = -\sin x$. [2 marks]

5. An angle α lies in quadrant IV with $\cos\alpha = \frac{5}{13}$. [4 marks]

(a) Find $\sin 2\alpha$. [2 marks]

(b) Find $\cos^2\alpha - \sin^2\alpha$ without calculating the angle. [2 marks]

6. Work with $u(x) = \sqrt{3}\cos x - \sin x$. [4 marks]

(a) Write u as one cosine with positive amplitude. [2 marks]

(b) Solve $u(x) = 0$ on $[0, 2\pi)$. [2 marks]

7. The function $v(x) = \csc x$ is defined only when $\sin x \neq 0$. [4 marks]

(a) State its vertical asymptotes and range. [2 marks]

(b) Explain why $\csc x = \frac{1}{2}$ has no real solution. [2 marks]

8. Study a proposed identity. [4 marks]

(a) Prove $\frac{1+\sin x}{\cos x} = \sec x + \tan x$ on the common domain. [2 marks]

(b) Evaluate $(\sec x + \tan x)(\sec x - \tan x)$ and state the same restriction. [2 marks]

9. A machine arm has horizontal coordinate $X(t) = 3\cos(\frac{\pi t}{4})$ cm, where time is in seconds. [4 marks]

(a) Find its period and the first time it reaches its minimum coordinate. [2 marks]

(b) Find every t in $[0, 8)$ for which $X(t) = 1.5$ cm. [2 marks]

10. A student says the amplitude of $6\sin(2x) + 8\cos(2x)$ is 14. [4 marks]

(a) Find the correct amplitude and explain the error. [2 marks]

(b) State the period and full range after adding a constant -3. [2 marks]