Further trigonometry - Practice test 2

10 questions | 40 marks | 65 minutes

Use representations, exact reasoning and graph restrictions. Supply exact answers where possible and state domains.

Equivalent correct methods earn the corresponding marks.

1. A point on the unit circle lies in quadrant III and has horizontal coordinate $-\frac{12}{13}$. [4 marks]

(a) Find its vertical coordinate. [2 marks]

Square and add to 1: the other coordinate has magnitude $\frac{5}{13}$.

Quadrant III makes it negative.

Answer: $-\frac{5}{13}$.

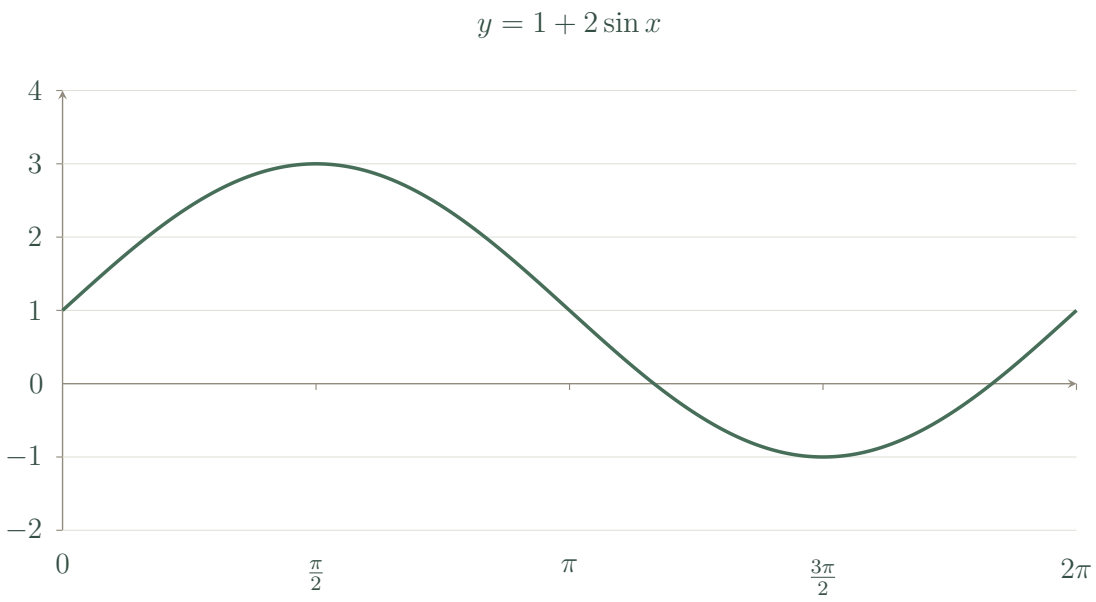
(b) Find $\tan\theta$ and $\cos(2\theta)$. [2 marks]

Tangent is $\frac{-\frac{5}{13}}{-\frac{12}{13}}$.

Double-angle cosine is $2(\frac{144}{169}) - 1 = \frac{119}{169}$.

Answer: $\frac{5}{12}$ and $\frac{119}{169}$.

2. The graph below shows $y = 1 + 2\sin x$. [4 marks]



(a) State the midline and range. [2 marks]

The amplitude is 2 around height 1.

Answer: Midline $y = 1$; range $[-1, 3]$.

(b) Give the coordinates of both turning points on $[0, 2\pi]$. [2 marks]

Sine is 1 at $\frac{\pi}{2}$ and -1 at $\frac{3\pi}{2}$.

Answer: Maximum $(\frac{\pi}{2}, 3)$; minimum $(\frac{3\pi}{2}, -1)$.

3. Solve equations on the interval $0 \le x \le 2\pi$. [4 marks]

(a) Solve $\cos(\frac{x}{2}) = 0$. [2 marks]

The argument is in $[0, \pi]$, where cosine vanishes only at $\frac{\pi}{2}$.

Answer: $x = \pi$.

(b) Solve $\sin^2 x = \frac{1}{4}$. [2 marks]

Take both signs: $\sin x = \pm\frac{1}{2}$.

Neither endpoint is a solution.

Answer: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

4. Develop exact angle values. [4 marks]

(a) Find $\tan 15^\circ$ exactly. [2 marks]

Use $\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$ and rationalise.

Answer: $2 - \sqrt{3}$.

(b) Use the sum formula to show $\sin(\pi + x) = -\sin x$. [2 marks]

$\sin \pi \cos x + \cos \pi \sin x = 0 - \sin x$.

Answer: Expand using $\sin \pi = 0$ and $\cos \pi = -1$.

5. An angle α lies in quadrant IV with $\cos\alpha = \frac{5}{13}$. [4 marks]

(a) Find $\sin 2\alpha$. [2 marks]

Sine is $-\frac{12}{13}$; use $2\sin\alpha\cos\alpha$.

Answer: $-\frac{120}{169}$.

(b) Find $\cos^2\alpha - \sin^2\alpha$ without calculating the angle. [2 marks]

Use $\frac{25-144}{169}$ or $2\cos^2\alpha - 1$.

Answer: $-\frac{119}{169}$.

6. Work with $u(x) = \sqrt{3}\cos x - \sin x$. [4 marks]

(a) Write u as one cosine with positive amplitude. [2 marks]

Expand: $2\cos x(\frac{\sqrt{3}}{2}) - 2\sin x(\frac{1}{2})$.

Answer: $2\cos(x + \frac{\pi}{6})$.

(b) Solve $u(x) = 0$ on $[0, 2\pi)$. [2 marks]

Cosine zero requires $x + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$ in the appropriate shifted interval.

Answer: $x = \frac{\pi}{3}, \frac{4\pi}{3}$.

7. The function $v(x) = \csc x$ is defined only when $\sin x \neq 0$. [4 marks]

(a) State its vertical asymptotes and range. [2 marks]

Zeros of the sine denominator give asymptotes; nonzero sine has magnitude at most 1.

Answer: Asymptotes $x = k\pi$; range $(-\infty, -1] \cup [1, \infty)$.

(b) Explain why $\csc x = \frac{1}{2}$ has no real solution. [2 marks]

Sine never exceeds 1, so the reciprocal equation is impossible.

Answer: It would require $\sin x = 2$.

8. Study a proposed identity. [4 marks]

(a) Prove $\frac{1+\sin x}{\cos x} = \sec x + \tan x$ on the common domain. [2 marks]

Require $\cos x \neq 0$, then use the definitions.

Answer: Split the fraction into $\frac{1}{\cos x} + \frac{\sin x}{\cos x}$.

(b) Evaluate $(\sec x + \tan x)(\sec x - \tan x)$ and state the same restriction. [2 marks]

Use difference of squares and $\sec^2 x - \tan^2 x = 1$.

Answer: 1 for $\cos x \neq 0$.

9. A machine arm has horizontal coordinate $X(t) = 3\cos(\frac{\pi t}{4})$ cm, where time is in seconds. [4 marks]

(a) Find its period and the first time it reaches its minimum coordinate. [2 marks]

The period is $\frac{2\pi}{\frac{\pi}{4}} = 8$; cosine reaches -1 at argument π.

Answer: Period 8 s; first minimum at 4 s.

(b) Find every t in $[0, 8)$ for which $X(t) = 1.5$ cm. [2 marks]

Cosine equals $\frac{1}{2}$ at arguments $\frac{\pi}{3}$ and $\frac{5\pi}{3}$; multiply by $\frac{4}{\pi}$.

Answer: $t = \frac{4}{3}, \frac{20}{3}$ s.

10. A student says the amplitude of $6\sin(2x) + 8\cos(2x)$ is 14. [4 marks]

(a) Find the correct amplitude and explain the error. [2 marks]

Same-frequency combination gives $\sqrt{36 + 64} = 10$.

Answer: 10, because the two terms do not peak together.

(b) State the period and full range after adding a constant -3. [2 marks]

Frequency 2 gives period π; translate $[-10, 10]$ down 3.

Answer: Period π; range $[-13, 7]$.