Further trigonometry - Practice test 1

10 questions | 42 marks | 65 minutes

Build from exact angles and graph features to identities and periodic applications. Angles are in radians unless a degree symbol is shown.

Equivalent correct methods earn the corresponding marks.

1. Work with radian measure. [4 marks]

(a) Convert 300° to radians. [2 marks]

$\frac{300\pi}{180} = \frac{5\pi}{3}$.

Answer: $\frac{5\pi}{3}$.

(b) A sector has radius 4 cm and angle $\frac{5\pi}{3}$. Find its area. [2 marks]

$A = \frac{1}{2}(4^2)(\frac{5\pi}{3}) = \frac{40\pi}{3}$.

Answer: $\frac{40\pi}{3}$ cm^2.

2. Consider $f(x) = 2 - 3\cos(2x)$. [4 marks]

(a) State its amplitude, period and range. [2 marks]

Use $|A| = 3$, $T = \frac{2\pi}{2}$ and midline 2.

Answer: Amplitude 3; period π; range $[-1, 5]$.

(b) Find the first maximum at $x \geq 0$. [2 marks]

A maximum needs $\cos 2x = -1$, first at $2x = \pi$.

Answer: $x = \frac{\pi}{2}$, $f(x) = 5$.

3. Solve and check the solution interval. [4 marks]

(a) Solve $\sin x = -\frac{1}{2}$ on $[0, 2\pi)$. [2 marks]

Sine is negative in quadrants III and IV; reference angle $\frac{\pi}{6}$.

Answer: $\frac{7\pi}{6}, \frac{11\pi}{6}$.

(b) Give all real solutions of $\tan(2x) = \sqrt{3}$. [2 marks]

$2x = \frac{\pi}{3} + k\pi$; divide by 2.

Answer: $x = \frac{\pi}{6} + \frac{k\pi}{2}$, $k \in \mathbb{Z}$.

4. Use compound-angle formulas. [4 marks]

(a) Find $\cos 15^\circ$ exactly. [2 marks]

Use $\cos(45^\circ - 30^\circ) = \cos 45 \cos 30 + \sin 45 \sin 30$.

Answer: $\frac{\sqrt{6}+\sqrt{2}}{4}$.

(b) For acute A, B with $\sin A = \frac{4}{5}$ and $\cos B = \frac{5}{13}$, find $\sin(A - B)$. [2 marks]

$\cos A = \frac{3}{5}$, $\sin B = \frac{12}{13}$.

Compute $(\frac{4}{5})(\frac{5}{13}) - (\frac{3}{5})(\frac{12}{13}) = -\frac{16}{65}$.

Answer: $-\frac{16}{65}$.

5. Let $\tan\theta = \frac{3}{4}$ with θ acute. [4 marks]

(a) Find $\sin 2\theta$ and $\cos 2\theta$. [2 marks]

Use a 3-4-5 triangle: sine $\frac{3}{5}$, cosine $\frac{4}{5}$.

Then $2(\frac{3}{5})(\frac{4}{5}) = \frac{24}{25}$ and $\frac{16}{25} - \frac{9}{25} = \frac{7}{25}$.

Answer: $\frac{24}{25}$ and $\frac{7}{25}$.

(b) Find $\tan 2\theta$. [2 marks]

Divide the double-angle sine by cosine, or use $\frac{2(\frac{3}{4})}{1 - \frac{9}{16}}$.

Answer: $\frac{24}{7}$.

6. Let $g(x) = 5 + 12\sin x + 5\cos x$. [4 marks]

(a) Express the varying part as one sine, specifying an acute phase. [2 marks]

Amplitude $\sqrt{144 + 25} = 13$; match cosine of phase $\frac{12}{13}$ and sine $\frac{5}{13}$.

Answer: $13\sin(x + \phi)$, $\phi = \tan^{-1}(\frac{5}{12})$.

(b) State the range of g. [2 marks]

Translate the interval $[-13, 13]$ by 5.

Answer: $[-8, 18]$.

7. Consider reciprocal trigonometry. [4 marks]

(a) Find all excluded inputs for $y = \sec(3x)$. [2 marks]

Exclude $3x = \frac{\pi}{2} + k\pi$.

Answer: $x = \frac{\pi}{6} + \frac{k\pi}{3}$.

(b) Solve $\csc x = -2$ on $[0, 2\pi)$. [2 marks]

Equivalent to $\sin x = -\frac{1}{2}$; neither solution is excluded.

Answer: $\frac{7\pi}{6}, \frac{11\pi}{6}$.

8. Prove an identity and preserve its domain. [5 marks]

(a) Show $\frac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$. [3 marks]

Use numerator $2\sin^2 x$ and denominator $2\cos^2 x$; divide.

Answer: The ratio is $\tan^2 x$ wherever $\cos x \neq 0$.

(b) State every excluded input from the original ratio. [2 marks]

$1 + \cos 2x = 2\cos^2 x$ is zero exactly at those inputs.

Answer: $x = \frac{\pi}{2} + k\pi$.

9. A model temperature is $T(t) = 18 + 6\cos(\frac{\pi(t-14)}{12})$ degrees Celsius, where t is hours after midnight. [4 marks]

(a) Find the maximum and minimum temperatures and their first occurrence in $[0, 24)$. [2 marks]

Cosine is 1 at 14; it is -1 half a 24-hour period earlier at 2.

Answer: Maximum 24 at $t = 14$; minimum 12 at $t = 2$.

(b) Find $T(8)$ and explain one modelling assumption. [2 marks]

The argument at 8 is $-\frac{\pi}{2}$, whose cosine is 0.

Weather may change the actual cycle.

Answer: $18\,^{\circ}\text{C}$; the model assumes a fixed repeating daily cycle.

10. Solve an equation without dividing away a factor. [5 marks]

(a) Solve $2\sin x\cos x = \sin x$ on $[0, 2\pi)$. [3 marks]

Factor $\sin x(2\cos x - 1) = 0$.

Sine zero gives 0 and π; cosine $\frac{1}{2}$ gives the other two.

Answer: $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$.

(b) Explain what would be lost by immediately dividing by $\sin x$. [2 marks]

At these values the factor being divided by is zero, although the original equation is true.

Answer: The valid solutions 0 and π.