Periodic modelling

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. A model has maximum $28\,^\circ$C and minimum $16\,^\circ$C. Find amplitude and midline. [2 marks]

Half-difference is 6; average is 22.

Answer: Amplitude $6\,^\circ$C; midline $22\,^\circ$C.

2. A wheel has radius 8 m, axle 10 m above ground and period 40 s. It starts at the bottom. Give a height rule. [2 marks]

The midline is 10, amplitude 8 and angular frequency $\frac{\pi}{20}$; negative cosine begins at height 2.

Answer: $h(t) = 10 - 8\cos(\frac{\pi t}{20})$ m.

3. For that wheel, find the height after 10 s. [2 marks]

The argument is $\frac{\pi}{2}$, whose cosine is 0.

Answer: 10 m.

4. For that wheel, find every time at height 14 m on $[0, 40)$. [2 marks]

Solve cosine $-\frac{1}{2}$, giving arguments $\frac{2\pi}{3}$, $\frac{4\pi}{3}$.

Multiply by $\frac{20}{\pi}$.

Answer: $t = \frac{40}{3}, \frac{80}{3}$ s.

5. Why is $\cos(\frac{2\pi t}{12})$ inappropriate for a 365-day daylight cycle when t is in days? [2 marks]

A 365-day cycle needs angular frequency $\frac{2\pi}{365}$ per day.

Answer: It repeats after 12 days.

6. Give two limitations of modelling a tide with a fixed sine wave. [2 marks]

The fixed amplitude/period assumptions ignore those effects, so predictions must be checked.

Answer: Weather can change heights; real successive tidal cycles can have unequal highs or timing.