Verifying identities

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Prove $\sec^2 x - \tan^2 x = 1$ wherever defined. [2 marks]

For $\cos x \neq 0$, $\tan^2 x + 1 = \sec^2 x$.

Answer: Divide the Pythagorean identity by $\cos^2 x$ and rearrange.

2. Show $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$ where both sides are defined. [2 marks]

On the common domain $\sin x \neq 0$, multiply numerator/denominator by $1 - \cos x$.

Use $1 - \cos^2 x = \sin^2 x$.

The original left side alone is also defined at some additional points, such as 0.

Answer: It follows by multiplying by the conjugate and cancelling.

3. Prove $(\sin x + \cos x)^2 = 1 + \sin 2x$. [2 marks]

The left side is $\sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + \sin 2x$.

Answer: Expand, combine squares and use double-angle sine.

4. Simplify $\frac{\cos^2 x}{1-\sin x}$, preserving restrictions. [2 marks]

Factor the numerator as $(1 - \sin x)(1 + \sin x)$ and cancel the nonzero common factor.

Answer: $1 + \sin x$, for $\sin x \neq 1$.

5. Disprove $\tan(A + B) = \tan A + \tan B$ using $A = B = \frac{\pi}{6}$. [2 marks]

Every relevant tangent exists at this pair, so it is a valid counterexample.

Answer: The sides are $\sqrt{3}$ and $\frac{2}{\sqrt{3}}$, which differ.

6. Prove $(\sec x - \tan x)(\sec x + \tan x) = 1$. [2 marks]

For $\cos x \neq 0$, the product equals $\sec^2 x - \tan^2 x = 1$.

Answer: Apply difference of squares and the secant identity.