Expressing a sine/cosine combination as one sinusoid

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find the amplitude of $\sin x + \cos x$. [2 marks]

Take the positive square root of $1^2 + 1^2$.

Answer: $\sqrt{2}$.

2. Write $\sin x + \cos x$ as a single sine. [2 marks]

Both matched coefficients are $\sqrt{2}(\frac{\sqrt{2}}{2}) = 1$.

Answer: $\sqrt{2}\sin(x + \frac{\pi}{4})$.

3. Write $4\sin x - 3\cos x$ as $R\sin(x + \phi)$ with $-\frac{\pi}{2} < \phi < 0$. [2 marks]

$R = 5$, cosine of phase $\frac{4}{5}$ and sine of phase $-\frac{3}{5}$.

Answer: $5\sin(x - \tan^{-1}(\frac{3}{4}))$.

4. Find the range of $2 + \sqrt{3}\sin x + \cos x$. [2 marks]

Amplitude is $\sqrt{3+1} = 2$ around midline 2.

Answer: $[0, 4]$.

5. Solve $\sin x + \cos x = 0$ on $[0, 2\pi)$. [2 marks]

Use $\sqrt{2}\sin(x + \frac{\pi}{4}) = 0$, subtract $\frac{\pi}{4}$ and restrict the interval.

Answer: $\frac{3\pi}{4}, \frac{7\pi}{4}$.

6. Why does the method not generally combine $a\sin x + b\cos 2x$ into one sine? [2 marks]

The expansion of a shifted sine produces sine and cosine terms with the same argument frequency.

Answer: The frequencies differ.