Compound-angle identities

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\sin 15^\circ$ exactly. [2 marks]

Expand $\sin(45^\circ - 30^\circ)$.

Answer: $\frac{\sqrt{6}-\sqrt{2}}{4}$.

2. Find $\cos 75^\circ$ exactly. [2 marks]

Expand $\cos(45^\circ + 30^\circ)$.

Answer: $\frac{\sqrt{6}-\sqrt{2}}{4}$.

3. Find $\tan 75^\circ$ exactly. [2 marks]

Use $\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$, then rationalise.

Answer: $2+\sqrt{3}$.

4. For acute A, B with $\sin A = \frac{5}{13}$ and $\sin B = \frac{3}{5}$, find $\sin(A+B)$. [2 marks]

The missing cosines are $\frac{12}{13}$ and $\frac{4}{5}$.

Add $\frac{20}{65} + \frac{36}{65}$.

Answer: $\frac{56}{65}$.

5. Given $\tan A = 2$, $\tan B = \frac{1}{3}$, calculate $\tan(A-B)$. [2 marks]

$\frac{2-\frac{1}{3}}{1+\frac{2}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$.

Answer: 1.

6. Disprove $\cos(A+B) = \cos A + \cos B$. [2 marks]

Both sides are defined, and the unequal values disprove the universal claim.

Answer: Set $A = B = 0$: the two sides are 1 and 2.