Solving trig equations and describing all solutions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $\cos x = -\frac{1}{2}$ on $[0, 2\pi)$. [2 marks]

Reference angle $\frac{\pi}{3}$; use quadrants II and III.

Answer: $\frac{2\pi}{3}, \frac{4\pi}{3}$.

2. Solve $\sin x = 0$ on $[-\pi, \pi]$. [2 marks]

All integer multiples of π in the closed interval count.

Answer: $-\pi, 0, \pi$.

3. Give all real solutions of $\cos x = 1$. [2 marks]

Cosine is 1 at complete turns.

Answer: $x = 2k\pi$, $k \in \mathbb{Z}$.

4. Solve $2\cos^2 x - \cos x - 1 = 0$ on $[0, 2\pi)$. [2 marks]

Factor $(2\cos x + 1)(\cos x - 1) = 0$; solve cosine $-\frac{1}{2}$ or 1.

Answer: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

5. Solve $\sin(2x) = 0$ on $[0, \pi]$. [2 marks]

$2x = k\pi$ with $k = 0, 1, 2$.

Answer: $0, \frac{\pi}{2}, \pi$.

6. Solve $\sin x(2\cos x - 1) = 0$ on $[0, 2\pi)$. [2 marks]

Sine zero gives 0 and π; cosine $\frac{1}{2}$ gives the other two.

Answer: $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$.