Proof and circle geometry - Reasoning test

Name: _____ Date: _____

10 questions | 52 marks | 75 minutes

Explain why methods work, judge the information in a diagram, and combine algebra with circle properties.

No calculator needed.

1. For an integer n, compare "n is divisible by 12" with "n is divisible by 6". [6 marks]

(a) Is divisibility by 12 necessary, sufficient, both or neither for divisibility by 6? [3 marks]

(b) Complete and justify: n is divisible by 12 if and only if n is divisible by 3 and ___. [3 marks]

2. For positive integers n, consider $n^2 + n + 17$. [4 marks]

(a) Find a positive-integer value for which this expression is not prime. [2 marks]

(b) Explain the logical effect of this one result, even if many earlier values were prime. [2 marks]

3. Let n be any integer. [6 marks]

(a) Prove $n^2 + n$ is even without listing individual n values. [3 marks]

(b) Prove that if n is odd, then n^2 is odd. [3 marks]

4. Consider $\sqrt{2}$ and $\sqrt{8}$. [5 marks]

(a) Simplify their sum and justify that it is irrational. [3 marks]

(b) Find their product and state whether that product is rational. [2 marks]

5. Let $A = (-\infty, -1] \cup [3, \infty)$ and $B = (-4, 5)$. [5 marks]

(a) Find $A \cap B$. [3 marks]

(b) Which of $-4, -1, 3, 5$ belong to the intersection? [2 marks]

6. A, B, C, D are distinct points in that cyclic order on a circle. Angle ABD is 32° and angle BAD is 68°. [4 marks]

(a) Find angle ACD. [2 marks]

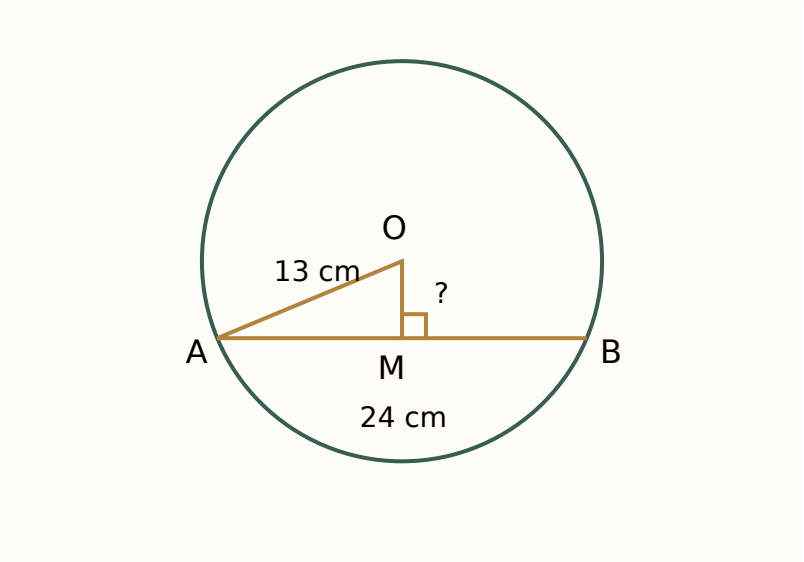
(b) Find angle BCD. [2 marks]

7. A convex quadrilateral has adjacent angles $A = 70^\circ$ and $B = 110^\circ$. [5 marks]

(a) Does this information alone prove the quadrilateral is cyclic? Give a reason or a counterexample. [3 marks]

(b) If C is also 110°, prove the quadrilateral is cyclic. [2 marks]

8. A circle has radius 13 cm. Chord AB is 24 cm long and chord CD is 10 cm long. [6 marks]



(a) Find each chord's distance from the centre. [3 marks]

(b) Explain generally why the longer of two chords in one circle lies closer to its centre. [3 marks]

9. Two chords meet at E. On AB, $AE = 2x$ and $EB = x$; on CD, $CE = 4$ and $ED = 8$. All lengths use the same unit. [5 marks]

(a) Find x and the length AB. [3 marks]

(b) Compare the distances of AB and CD from the centre. [2 marks]

10. Triangles ABC and DEF have respective side lengths $AB = 8$, $BC = 10$, $AC = 12$ and $DE = 12$, $EF = 15$, $DF = 18$. [6 marks]

(a) Prove they are similar, specifying the correspondence. [3 marks]

(b) Find the area ratio $DEF : ABC$ without needing to calculate either absolute area. [3 marks]