Proof and circle geometry - Applications test

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

Use logical claims, exact arithmetic and circle theorems to analyse practical designs and identify errors.

No calculator needed.

1. A numerical checker accepts an integer exactly when it is divisible by both 3 and 5. [5 marks]

(a) Express the rule as an equivalence using divisibility by one integer. [3 marks]

(b) Does acceptance imply divisibility by 30? Give evidence. [2 marks]

2. An algorithm assumes $a < b$ always implies $a^2 < b^2$. [5 marks]

(a) Give a counterexample using negative integers. [2 marks]

(b) Prove the implication is valid under the extra condition $0 \leq a < b$. [3 marks]

3. A student says: “If $x+y$ is rational, then x and y must each be rational.” [5 marks]

(a) Disprove this implication. [2 marks]

(b) Prove that if x is rational and $x+y$ is rational, then y is rational. [3 marks]

4. Prove that $\sqrt{3}$ is irrational. [7 marks]

(a) Show an integer not divisible by 3 has a square not divisible by 3. [3 marks]

(b) Use a fraction in lowest terms to complete a contradiction proof. [4 marks]

5. A formula $f(x)=\frac{1}{x-2}$ is to be used only when $-1 \leq x < 5$. [5 marks]

(a) Write its permitted input set in interval notation. [3 marks]

(b) Explain why $[-1, 5)$ alone is incorrect. [2 marks]

6. Two camera positions P and Q lie on opposite segments of chord AB of a circular arena. The minor central angle AOB is 84°. P lies on the major arc and Q on the minor arc. [5 marks]

(a) Find the viewing angle APB. [2 marks]

(b) Find AQB and explain why the two angles differ. [3 marks]

7. A four-sided frame $ABCD$ is designed to fit a circular opening. In cyclic order its angles A and C are $2x+10$ and $3x-5$ degrees, and B is 110°. [6 marks]

(a) Find x, A and C. [3 marks]

(b) Find the exterior angle at D formed by extending one side. [3 marks]

8. A circular pond has radius 9 m. A post P is 15 m from its centre. Two straight cables from P just touch the pond at A and B. [5 marks]

(a) Find the length of cable PA. [3 marks]

(b) Find the total length of the two cables and justify the equality you use. [2 marks]

9. A survey line from P crosses a circle at A and B, with $PA = 6$ m and $AB = 18$ m. A tangent from P touches at T. [6 marks]

(a) Find PT. [3 marks]

(b) A second survey line has outside portion 9 m. Find its portion inside the circle. [3 marks]

10. A circular skylight has centre O and a non-diameter chord AB. M is the midpoint of AB. [7 marks]

(a) Prove OM is perpendicular to AB. [4 marks]

(b) If the radius is 10 cm and AB is 16 cm, find OM. [3 marks]