Proof and circle geometry - Practice test 2

Name: _____ Date: _____

10 questions | 52 marks | 75 minutes

A second balanced paper emphasising converse arguments, divisibility, constructions and similarity.

No calculator needed.

1. For a real number x, consider the statement $x = 3 \Rightarrow x^2 = 9$. [5 marks]

(a) Explain why the implication is true. [2 marks]

(b) Write a corrected equivalence for $x^2 = 9$. [3 marks]

2. Consider two geometric claims. [5 marks]

(a) Disprove: every quadrilateral with a pair of parallel sides is a parallelogram. [3 marks]

(b) Disprove: every rhombus is a square. [2 marks]

3. Let n be an integer. [5 marks]

(a) Prove $n(n+1)$ is even. [2 marks]

(b) Prove $n(n+1)(n+2)$ is divisible by 6. [3 marks]

4. Let r be a non-zero rational number and x be irrational. [5 marks]

(a) Prove rx is irrational. [3 marks]

(b) Explain why the condition $r \neq 0$ is essential. [2 marks]

5. Describe solution sets over the reals. [4 marks]

(a) Solve $x^2 \leq 16$ and write an interval. [2 marks]

(b) Find $[-4, 4] \cap (2, \infty)$. [2 marks]

6. AB is a diameter and P is another point on its circle. AP is 7 cm and PB is 24 cm. [5 marks]

(a) Find AB. [3 marks]

(b) A student calls AP the hypotenuse. Explain the error using the angle location. [2 marks]

7. $ABCD$ is an isosceles trapezium with AB parallel to CD, and base angles A and B equal. [5 marks]

(a) Explain why angle A plus angle D is 180°. [2 marks]

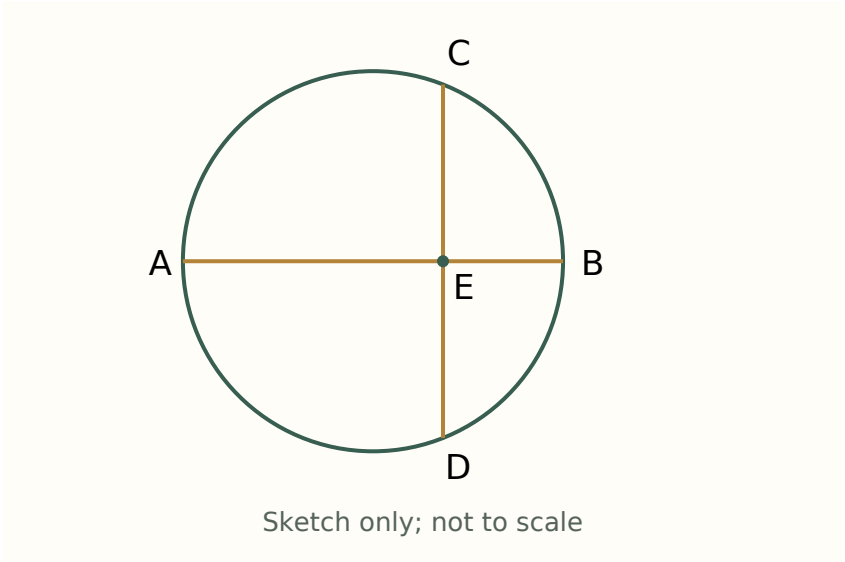
(b) Prove the trapezium is cyclic. [3 marks]

8. Tangents PA and PB touch a circle with centre O. The angle APB is 70°. [6 marks]

(a) Find angle AOB. [3 marks]

(b) Explain why OP bisects angle APB. [3 marks]

9. Chords AB and CD meet at E. $AE = x$, $EB = x + 5$, $CE = 3$ and $ED = 12$, in centimetres. [5 marks]



Sketch only; not to scale

(a) Form and solve an equation for x. [3 marks]

(b) State the permitted x and find AB. [2 marks]

10. In a right triangle ABC, angle A is right and altitude AH meets BC. $BH = 9$ cm and $HC = 16$ cm. [7 marks]

(a) Find AH using the geometric-mean relationship. [3 marks]

(b) Find AB and AC, explaining which hypotenuse piece belongs to each. [4 marks]