Congruence and similarity in geometric proofs

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Triangles ABC and DEF have $AB = DE$, $BC = EF$ and $AC = DF$. State a valid conclusion with a test. [2 marks]

All three corresponding sides are equal.

Answer: $\triangle ABC \cong \triangle DEF$ by SSS.

2. A student proves all three corresponding angles equal and claims congruence. Give a concrete counterexample. [2 marks]

All corresponding angles are $60°$, but their side lengths differ.

Answer: Equilateral triangles with side lengths 2 and 5.

3. Triangles ABC and DEF are similar in that order. $AB = 6$, $BC = 9$ and $DE = 10$. Find EF. [2 marks]

Scale from ABC to DEF is $\frac{10}{6} = \frac{5}{3}$; $EF = 9(\frac{5}{3}) = 15$.

Answer: 15 units.

4. AB and AC are tangents from A to a circle with centre O. Prove AO bisects angle BAC. [4 marks]

$OB = OC$ as radii; AO is a shared hypotenuse; tangent-radius angles B and C are right.

Corresponding angles BAO and OAC are equal.

Answer: Triangles ABO and ACO are congruent by RHS.

5. An altitude to the hypotenuse divides it into lengths 4 and 9. Find the altitude and explain the theorem used. [3 marks]

By similarity, $h^2 = 4 \times 9 = 36$. Take $h = 6 > 0$.

Answer: 6 units.

6. In the same altitude construction, the hypotenuse pieces are p and q. Explain why $p + q$ is not the altitude in general. [3 marks]

The geometric-mean theorem gives $h^2 = pq$.

For example, $p = 4$, $q = 9$ give hypotenuse 13 and altitude 6, not 13.

Answer: It is the full hypotenuse, whereas the altitude is $\sqrt{pq}$.