Rational/irrational numbers and simple number proofs

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. Prove that the sum of two rational numbers is rational. [2 marks]

$ad+bc$ and bd are integers and $bd \neq 0$.

Answer: $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.

2. Prove that a non-zero rational multiple of an irrational number is irrational. [3 marks]

Let $r \neq 0$ be rational and x irrational. If rx were rational, then $x=\frac{rx}{r}$ would be rational, a contradiction.

Answer: The product is irrational.

3. Show that $0.\overline{27}$ is rational. [3 marks]

Let $x=0.2727\ldots$. Then $100x-x=27$, so $x=\frac{27}{99}=\frac{3}{11}$.

Answer: $\frac{3}{11}$.

4. Give irrational numbers whose sum is rational, and another pair whose sum is irrational. [2 marks]

0 is rational. A non-zero rational multiple of $\sqrt{2}$ is irrational.

Answer: $\sqrt{2}+(-\sqrt{2})=0$; $\sqrt{2}+\sqrt{2}=2\sqrt{2}$.

5. Prove that $\sqrt{5}$ is irrational. [4 marks]

Assume $\sqrt{5}=\frac{a}{b}$ in lowest terms. Then $a^2=5b^2$.

A square divisible by the prime 5 has its root divisible by 5. Set $a=5k$.

Then $b^2=5k^2$, so 5 also divides b, contradicting lowest terms.

Answer: It is irrational.

6. Is the reciprocal of a non-zero irrational number irrational? Prove your answer. [3 marks]

If $\frac{1}{x}$ were rational and non-zero, its reciprocal x would be rational, a contradiction.

Answer: Yes.