Counterexamples

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Disprove: every even integer is divisible by 4. [2 marks]

6 is even, but $\frac{6}{4}$ is not an integer.

Answer: 6.

2. Disprove: for every real x, $\sqrt{x^2} = x$. [2 marks]

$\sqrt{(-3)^2} = 3 \neq -3$. The correct general identity is $\sqrt{x^2} = |x|$.

Answer: $x = -3$.

3. Disprove: the sum of any two irrational numbers is irrational. [2 marks]

Both numbers are irrational; their sum is 0, which is rational.

Answer: $\sqrt{2}$ and $-\sqrt{2}$.

4. Disprove: if $ab = 0$, then both a and b are zero. [2 marks]

The product is zero although $b \neq 0$. The valid conclusion is that at least one factor is zero.

Answer: $a = 0$, $b = 5$.

5. A claim concerns positive integers. Why does $n = 0$ not disprove it? [2 marks]

A counterexample must satisfy all of the original assumptions, including positivity.

Answer: 0 is outside the domain.

6. For integer n, the first four values of $n^2 - n + 5$ are prime. Find a positive-integer counterexample. [3 marks]

$5^2 - 5 + 5 = 25 = 5 \times 5$, so the universal prime claim is false.

Answer: $n = 5$, giving 25.