Marking guide | Probability

Probability - Applications test

16 questions | 48 marks | 60 minutes

Apply compound-event models to games, collections and evidence, then defend assumptions and conclusions.

Equivalent correct methods earn the corresponding marks.

1. A computer model gives success probability 0.3 on each of two independent days. Find $P(\text{success on both days})$. [2 marks]

$0.3 \times 0.3 = 0.09$ under the stated independence assumption.

Answer: 0.09

2. A lunch bundle offers 3 sandwiches, 2 drinks and 2 snacks. Every combination is equally likely. Find $P(\text{vegetarian sandwich and water})$ if exactly one sandwich is vegetarian and exactly one drink is water. [3 marks]

There are $3 \times 2 \times 2 = 12$ bundles.

The preferred sandwich/drink can be paired with either snack: two favourable bundles.

Answer: $\frac{1}{6}$

3. A box has five 10-dollar vouchers and three 20-dollar vouchers. Choose two at random without replacement. Find $P(\text{two } 20-\text{dollar vouchers})$. [2 marks]

$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$.

Answer: $\frac{3}{28}$

4. A game uses a fair die and an independent equal spinner labelled A and B. A player wins only with an even die result and A. [4 marks]

(a) Find the win probability. [2 marks]

$\frac{3}{6} \times \frac{1}{2} = \frac{1}{4}$.

Answer: $\frac{1}{4}$

(b) Predict the wins in 200 plays. Is this count guaranteed? [2 marks]

$200 \times \frac{1}{4} = 50$.

Random variation can change the observed count.

Answer: 50 expected wins; not guaranteed.

5. A tray has 8 good parts and 2 faulty parts. Choose two at random without replacement. Find $P(\text{no faulty parts})$. [2 marks]

$\frac{8}{10} \times \frac{7}{9} = \frac{56}{90} = \frac{28}{45}$.

Answer: $\frac{28}{45}$

6. A tray has 8 good parts and 2 faulty parts. Choose two at random without replacement. Find $P(\text{at least one faulty part})$. [3 marks]

The complement is two good parts, probability $\frac{28}{45}$.

$1 - \frac{28}{45} = \frac{17}{45}$.

Answer: $\frac{17}{45}$

7. Among 40 students, 22 used a bus this week, 16 used a bike and 6 used both. Find $P(\text{neither})$ for one randomly selected student. [3 marks]

At least one: $22 + 16 - 6 = 32$.

Neither: $40 - 32 = 8$, so $\frac{8}{40} = \frac{1}{5}$.

Answer: $\frac{1}{5}$

8. Among 40 students, 22 used a bus this week, 16 used a bike and 6 used both. One student is chosen at random. Find $P(\text{exactly one of bus or bike})$. [3 marks]

Bus only: 16; bike only: 10.

Exactly one: 26; $\frac{26}{40} = \frac{13}{20}$.

Answer: $\frac{13}{20}$

9. A drawer holds 4 blue and 2 red socks. Two are chosen at random without replacement. Find $P(\text{matching colours})$. [3 marks]

BB gives $\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$.

RR gives $\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$.

Add: $\frac{14}{30} = \frac{7}{15}$.

Answer: $\frac{7}{15}$

10. A two-stage game is won 36 times in 90 plays. Use these results to estimate wins in another 120 similar plays. [2 marks]

Estimated probability is $\frac{36}{90} = 0.4$.

$120 \times 0.4 = 48$.

Answer: About 48 wins.

11. A class simulation records 40 successes in 300 trials. A fair-model probability is $\frac{1}{8}$. Compare the observed and model rates. [3 marks]

$\frac{40}{300} = \frac{2}{15}$.

The small difference alone does not prove the model wrong.

Answer: Observed $\frac{2}{15} \approx 0.133$ versus model $\frac{1}{8} = 0.125$.

12. Group A simulates two draws with replacement; Group B simulates two draws without replacement. Why should their success counts not be pooled as an estimate for one unchanged game? [3 marks]

Replacement changes the second-stage collection.

First choose one game rule and combine only comparable trials.

Answer: They model different rules and can have different event probabilities.

13. Use independent equally likely random digits. Digits 0-3 mean S and 4-9 mean F. Read pairs 12, 74, 06, 39, 88, 45. [6 marks]

(a) Write the six outcomes and estimate $P(\text{exactly one S})$. [3 marks]

Two of six trials have exactly one S.

Answer: SS, FF, SF, SF, FF, FF; estimate $\frac{1}{3}$.

(b) Find the theoretical probability of exactly one S. [3 marks]

Each stage has success probability 0.4.

SF and FS have total probability $0.4 \times 0.6 + 0.6 \times 0.4 = 0.48$.

Answer: 0.48

14. A class correctly removes the first counter during each two-draw trial, but never restores the full bag between trials. Explain why this changes the intended simulation. [3 marks]

Keep removal within a trial.

Reset the full starting collection between complete trials.

Answer: Later trials start from a different bag, so they no longer repeat the same experiment.

15. A fair coin chooses bag A or bag B. A has 3 red and 1 blue counter; B has 1 red and 3 blue. Draw one counter at random from the chosen bag. Find $P(\text{red})$. [3 marks]

Red through A: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

Red through B: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

Add disjoint paths to get $\frac{1}{2}$.

Answer: $\frac{1}{2}$

16. A model predicts win probability $\frac{2}{5}$. In 200 plays there are 73 wins. Give a justified short conclusion and one useful next step. [3 marks]

Expected wins: $200 \times \frac{2}{5} = 80$.

Observed frequency: $\frac{73}{200} = 0.365$.

A seven-win difference is not, by itself, proof of a faulty model.

Answer: The observed rate 0.365 is below 0.4, but finite variation may explain the difference; check the rules and repeat more trials.