Testing/refining construction algorithms

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Test radius $r = 5$ cm on segment $AB = 10$ cm for the usual two-crossing bisector. [3 marks]

The chosen radius equals half the segment.

Two distinct crossings require more than half.

Answer: It fails: the circles touch once. Choose $r > 5$ cm.

2. A factor-2 enlargement rule doubles x but leaves y unchanged. Test $P(2, 3)$. [3 marks]

An enlargement from the origin scales both coordinates.

Changing only one coordinate distorts the figure.

Answer: It gives $(4, 3)$, but the correct image is $(4, 6)$.

3. Why is $P(4, 0)$ a poor only test for the faulty factor-2 rule $(x, y) \mapsto (2x, y)$? [3 marks]

Doubling zero leaves it zero.

Use a point with nonzero y, such as $(1, 2)$.

Answer: Both the faulty and correct rules give $(8, 0)$.

4. Test the midpoint rule $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ on $A(-4, 2)$, $B(2, 8)$. [2 marks]

Average x to get -1.

Average y to get 5.

Answer: $M(-1, 5)$.

5. A factor-$\frac{1}{2}$ rule rounds every image coordinate to a whole number. Test $P(1, 3)$ and refine the rule. [3 marks]

Whole-number rounding changes the required distance ratio and direction.

Keep exact halves or use a half-unit grid.

Answer: The exact image is $(0.5, 1.5)$; retain these values.

6. Give three useful types of test for a bisector construction and explain why tests alone are not a proof. [4 marks]

Choose inputs that expose different possible faults.

A general argument must explain why equal-radius crossings give the required line.

Answer: Vary segment length, direction and radii near the half-length boundary. Tests check selected cases only.