Quadratics and Parabolas - Applications

16 questions | 47 marks | 60 minutes

Applications using area, financial models, domains and model checks.

Equivalent correct methods earn the corresponding marks.

1. A square has side x cm. Write its area rule and find its area when $x = 5$. [2 marks]

Area is side times side.

$5^2 = 25$.

Answer: $A = x^2$; $25\ \text{cm}^2$

2. A rectangle has sides $x + 1$ cm and $x + 3$ cm. Expand its area rule. [2 marks]

$(x + 1)(x + 3) = x^2 + 3x + x + 3$.

Answer: $A = (x^2 + 4x + 3)\ \text{cm}^2$

3. A square poster has side $n + 2$ cm. Correct the area claim $A = n^2 + 4$. [2 marks]

$(n + 2)^2 = (n + 2)(n + 2)$.

The two cross-products total $4n$.

Answer: $A = (n^2 + 4n + 4)\ \text{cm}^2$

4. A garden is x m wide and $x + 4$ m long with area $45\ \text{m}^2$. Find its dimensions. [3 marks]

$x^2 + 4x - 45 = (x + 9)(x - 5) = 0$.

Reject -9 and use $x = 5$.

Answer: $5\ \text{m}$ by $9\ \text{m}$

5. Two consecutive positive whole numbers have product 72. Find them. [3 marks]

$n^2 + n - 72 = (n+9)(n-8) = 0.$

Only $n = 8$ is positive.

Answer: 8 and 9

6. A toy rocket has height $h = -t^2 + 4t + 5$ metres. Find its starting height and height after 1 second. [2 marks]

$h(0) = 5.$

$h(1) = -1 + 4 + 5 = 8.$

Answer: 5 m and 8 m

7. For $h = -t^2 + 4t + 5$, find when the rocket reaches the ground, with $t \geq 0$. [3 marks]

$t^2 - 4t - 5 = (t-5)(t+1) = 0.$

Reject $t = -1.$

Answer: 5 s

8. A toy rocket follows $h = -t^2 + 4t + 5$ metres for $t \geq 0$ seconds. Find its maximum height and when it occurs. [3 marks]

The roots $-1, 5$ have midpoint 2.

$h(2) = -4 + 8 + 5 = 9.$

Answer: 9 m at $t = 2$ s.

9. A ticket model is $R = n(16 - n)$ dollars. Find revenue for $n = 6$. [2 marks]

$6(16 - 6) = 6 \times 10 = 60.$

Answer: 60

10. Find both whole-number counts giving $R = 60$ in $R = n(16 - n)$. [3 marks]

$n^2 - 16n + 60 = (n - 6)(n - 10) = 0$.

Answer: $n = 6, 10$

11. Revenue is $R = n(16 - n)$ dollars. Counts $n = 6$ and 10 both give $R = 60$, but the venue takes at most 8 people. Which count is feasible? [2 marks]

10 exceeds the capacity 8, but 6 does not.

Answer: $n = 6$ only.

12. A square garden of side x m has a 1 m wide path around its outside. Total outer area is $100\ \text{m}^2$. [5 marks]

(a) Write an equation for x. [2 marks]

The path adds 1 m at each end of a side.

Answer: $(x + 2)^2 = 100$

(b) Find the garden side and the path area. [3 marks]

The positive outer side is 10 m, so $x = 8$.

$100 - 8^2 = 36$.

Answer: Side $8\ \text{m}$; path $36\ \text{m}^2$.

13. A model for a positive count is $n^2 + 2n - 35 = 0$. Find the feasible count. [3 marks]

$(n + 7)(n - 5) = 0$ gives $-7, 5$.

A positive count excludes -7.

Answer: $n = 5$

14. A graph experiment uses $y=a(x-2)^2+3$ with $a=1,2,-1$. Give the shared turn and each opening. [3 marks]

The bracket is zero at $x=2$ for every a.

The sign of a controls the opening.

Answer: All turn at $(2,3)$; $a=1,2$ open up; $a=-1$ opens down.

15. A model requires $x^2+2x=10$, $x>0$. Locate x between consecutive tenths. [3 marks]

$2.3^2+2(2.3)-10=-0.11$.

$2.4^2+2(2.4)-10=0.56$.

The continuous curve changes sign.

Answer: Between 2.3 and 2.4.

16. Quotes are $C_A=n^2+4n+30$ and $C_B=14n+14$ dollars for n items. [6 marks]

(a) Find the two counts where the quotes are equal. [3 marks]

$n^2-10n+16=(n-2)(n-8)=0$.

Answer: $n=2,8$

(b) Which is cheaper for 5 items, and by how much? [3 marks]

$C_A=25+20+30=75$.

$C_B=70+14=84$.

$84-75=9$.

Answer: A by 9.