Marking guide | Linear equations and graphs

Linear equations and graphs - applications paper

16 questions | 53 marks | 60 minutes

Use linear relationships, coordinate geometry and model checks in connected practical situations.

Equivalent correct methods earn the corresponding marks.

1. A group pays 12 dollars to reserve a lane and 6 dollars for each player. The bill is 84 dollars. How many players are there? [2 marks]

Let n be the player count: $12 + 6n = 84$.

Then $6n = 72$, so $n = 12$.

Answer: 12 players

2. A tank contains 20 L after 1 minute and 68 L after 5 minutes. Assume a constant filling rate. Find the rate. [2 marks]

The volume increases by $68 - 20 = 48$ L in $5 - 1 = 4$ minutes.

The rate is $\frac{48}{4} = 12$ L/min.

Answer: 12 L/min

3. A map grid is measured in kilometres. A shelter is at $(3, 1)$ and a gate is at $(9, 9)$. [4 marks]

(a) Find the midpoint of the direct route. [2 marks]

Average coordinates: $\frac{3+9}{2} = 6$ and $\frac{1+9}{2} = 5$.

Answer: $(6, 5)$

(b) Find the length of the direct route. [2 marks]

The coordinate changes are 6 km and 8 km.

$d = \sqrt{6^2 + 8^2} = 10$ km.

Answer: 10 km

4. A phone battery is modelled by $B = 90 - 7t$ percent after t hours, for $0 \le t \le 10$. When does the model give 55%? [3 marks]

$55 = 90 - 7t$, so $7t = 35$ and $t = 5$.

Five hours is within the stated range.

Answer: 5 hours

5. Two hire plans charge $A = 18 + 3h$ and $B = 6 + 5h$ dollars for a whole number of hours h, with $1 \le h \le 8$. [5 marks]

(a) Find the hire length at which the two plans cost the same. [3 marks]

Set $18 + 3h = 6 + 5h$.

Then $12 = 2h$, giving $h = 6$.

Substitution gives 36 dollars in either rule.

Answer: 6 hours; both cost 36 dollars.

(b) Which plan is cheaper for 4 hours and for 8 hours? [2 marks]

At 4 hours, $A = 30$ and $B = 26$.

At 8 hours, $A = 42$ and $B = 46$.

Answer: For 4 hours B is cheaper; for 8 hours A is cheaper.

6. A cable follows $2x + y = 8$. A second cable is perpendicular to it. Find the second cable's gradient. [2 marks]

The first cable is $y = -2x + 8$, with gradient -2.

Its negative reciprocal is $\frac{1}{2}$.

Answer: $\frac{1}{2}$

7. A straight fence passes through $(-2, 1)$ and $(2, 5)$. A marker is proposed at $(5, 8)$. Is the marker on the fence? Justify. [3 marks]

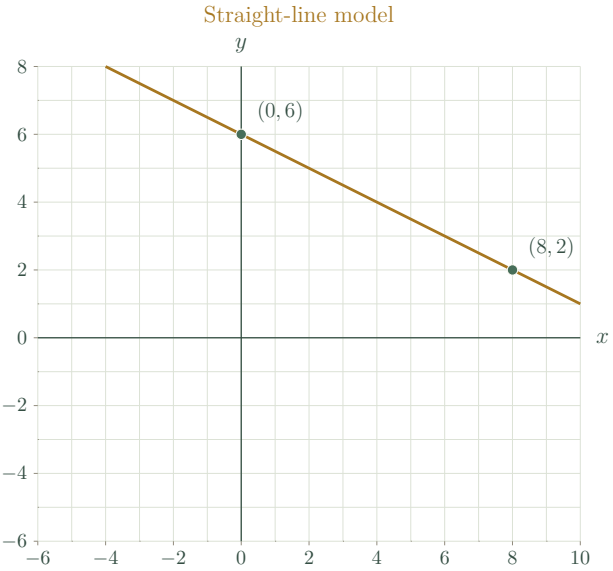
The gradient between the first two points is $\frac{5-1}{2+2} = 1$.

The line is $y = x + 3$.

At $x = 5$, the rule gives $y = 8$.

Answer: Yes.

8. The displayed straight-line model relates x and y. [5 marks]



(a) Find its gradient and equation using the labelled points. [3 marks]

From $(0, 6)$ to $(8, 2)$, rise $= -4$ and run $= 8$.

Thus $m = -\frac{1}{2}$; the vertical intercept is 6.

Answer: Gradient $-\frac{1}{2}$; $y = -\frac{1}{2}x + 6$.

(b) Use the equation to find when $y = 3$. [2 marks]

$3 = -\frac{x}{2} + 6$ gives $\frac{x}{2} = 3$, so $x = 6$.

Answer: $x = 6$

9. A path passes through $(4, -3)$ and is perpendicular to the vertical boundary $x = -1$. Write the path's equation. [2 marks]

A line perpendicular to a vertical line is horizontal.

It must have the fixed vertical coordinate -3.

Answer: $y = -3$

10. The midpoint of a bridge is $M(1, 2)$. One end is $A(-3, 5)$. Find the other end B. [3 marks]

$B = (2 \times 1 - (-3), 2 \times 2 - 5) = (5, -1)$.

Check the coordinate averages give $(1, 2)$.

Answer: $B = (5, -1)$

11. A plant is 12 cm high at day 2 and 30 cm high at day 8. Find the average gradient of height against time, including units. [2 marks]

$m = \frac{30 - 12}{8 - 2} = \frac{18}{6} = 3$ cm/day.

Units describe height change per day.

Answer: 3 cm/day

12. A club recorded the costs in the table. Can one constant-rate linear model fit all three entries? Explain. [3 marks]

Recorded club costs

People	2	4	6
Cost (dollars)	18	28	42

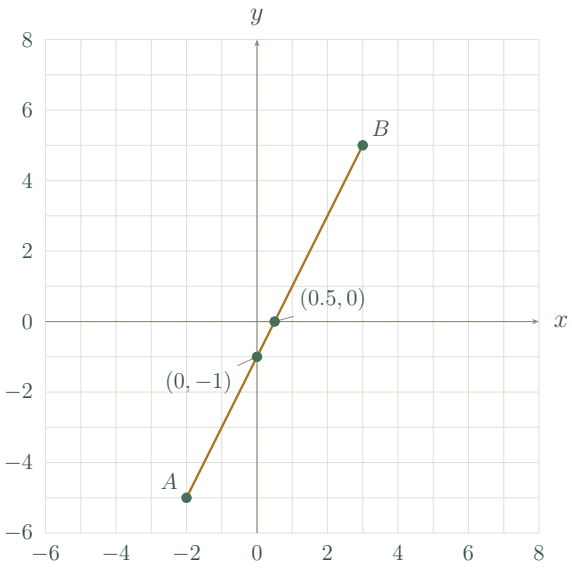
From 2 to 4 people, the cost rises 10 dollars for 2 people, or 5 dollars each.

From 4 to 6 people, it rises 14 dollars for 2 people, or 7 dollars each.

The rates differ.

Answer: No.

13. On the supplied grid, draw $y = 2x - 1$ for $-2 \le x \le 3$. Label both axis intercepts. [4 marks]



At $x = -2$, $y = -5$; at $x = 3$, $y = 5$.

The vertical intercept is $(0, -1)$.

Set $y = 0$: $2x - 1 = 0$, so the horizontal intercept is $(0.5, 0)$.

Join the points with one straight segment.

Answer: A straight segment through $(-2, -5)$ and $(3, 5)$; intercepts $(0, -1)$ and $(0.5, 0)$.

14. A student says that changing $y = 2x + 1$ to $y = 5x + 1$ shifts the graph straight upwards. Explain why this is wrong. [3 marks]

At $x = 0$, both rules give 1.

The output difference is $3x$, which changes with x.

A vertical shift would add the same amount at every input.

Answer: The intercept stays $(0, 1)$ while the gradient changes, so the line becomes steeper rather than moving up by a fixed amount.

15. A line of gradient -2 passes through $(3, 4)$. Find where it crosses the horizontal axis. [3 marks]

First find the rule: $4 = -2(3) + c$, so $c = 10$.

Then $0 = -2x + 10$, giving $x = 5$.

Answer: $(5, 0)$

16. A straight walking track joins $A(0, 0)$ and $B(8, 6)$ on a kilometre grid. A rest point M is halfway along it. A straight service path passes through M at right angles to AB. [7 marks]

(a) Find M and the gradient of AB. [2 marks]

Average the endpoint coordinates.

The gradient is $\frac{6}{8} = \frac{3}{4}$.

Answer: $M = (4, 3)$; gradient $\frac{3}{4}$.

(b) Find an equation for the service path. [3 marks]

The perpendicular gradient is $-\frac{4}{3}$.

Use M: $3 = (-\frac{4}{3})(4) + c$, so $c = \frac{25}{3}$.

Answer: $y = -\frac{4}{3}x + \frac{25}{3}$

(c) How far is M from A along the walking track? [2 marks]

$AM = \sqrt{4^2 + 3^2} = 5$ km.

Answer: 5 km