Marking guide | Probability

Probability - Applications test

16 questions | 47 marks | 60 minutes

Apply probability and evaluate simulation choices.

Equivalent correct methods earn the corresponding marks.

1. A uniform random four-digit code may begin with zero and may repeat digits. How many possible codes are there? [2 marks]

Ten choices in each of four positions give 10^4.

Answer: 10,000.

2. A four-digit code is chosen uniformly from digits 0-9. Repeats and a leading zero are allowed. Find $P(\text{the code is } 0000)$. [2 marks]

Exactly one of the 10,000 equally likely codes is 0000.

Answer: $\frac{1}{10,000}$.

3. Six volunteers fill two different roles. Count the possible assignments. [2 marks]

6 choices for the first role and 5 for the second.

Answer: 30.

4. Six volunteers form a two-person group with no different roles. Count the groups and explain why this differs from assigning two different roles to six people. [3 marks]

The 30 ordered selections count each group twice.

Divide by 2 because exchanging the people does not change the group.

Answer: 15.

5. Two counters are drawn from 3 red and 2 blue. [4 marks]

(a) Find $P(\text{two red})$ without replacement. [2 marks]

$(\frac{3}{5})(\frac{2}{4}) = \frac{3}{10}$.

Answer: $\frac{3}{10}$.

(b) Find $P(\text{two red})$ with replacement. [2 marks]

$(\frac{3}{5})^2 = \frac{9}{25}$.

Answer: $\frac{9}{25}$.

6. Using the club table, find the probability a chosen student belongs to neither club. [2 marks]

Invented membership data for 60 students.

Music membership	Drama	Not drama	Total
Music	8	12	20
Not music	10	30	40
Total	18	42	60

The neither count is 30 out of 60.

Answer: $\frac{1}{2}$.

7. A raffle has 50 student tickets,30 teacher tickets and 20 community tickets. Draw two tickets without replacement. Find $P(\text{both student})$. [3 marks]

$(\frac{50}{100})(\frac{49}{99}) = \frac{2450}{9900} = \frac{49}{198}$.

Answer: $\frac{49}{198}$.

8. A raffle has 50 student tickets, 30 teacher tickets and 20 community tickets. Two tickets are drawn uniformly without replacement. Find $P(\text{exactly one teacher ticket})$. [3 marks]

Teacher then non-teacher:$(\frac{30}{100})(\frac{70}{99}) = \frac{7}{33}$.

The reverse order also gives $\frac{7}{33}$;total $\frac{14}{33}$.

Answer: $\frac{14}{33}$.

9. Two fair dice are rolled. Given at least one die shows 6, find $P(\text{sum } 7)$. [3 marks]

There are 11 ordered pairs with at least one 6.

Only $(1,6)$ and $(6,1)$ sum to 7.

Answer: $\frac{2}{11}$.

10. A simulation uses independent uniform random digits 0-9, with 0-5 meaning success. In 20 digits there are 13 successes. Estimate the success probability and compare it with the model probability. [3 marks]

$\frac{13}{20} = 0.65$.

The observed estimate is 0.05 higher; variation is expected.

Answer: Estimate 0.65; model probability 0.6.

11. A simulation generates sums 2-12 uniformly and reports $P(\text{sum } 7)$ near $\frac{1}{11}$. Explain why this is not a fair two-dice simulation. [3 marks]

Two dice have 36 ordered pairs,6 of which sum to 7.

Generate the two die values independently and then add.

Answer: The generator gives each sum equal probability, unlike two dice.

12. $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$. [4 marks]

(a) Find $P(A \mid B)$. [2 marks]

$\frac{0.4}{0.5} = 0.8$.

Answer: 0.8.

(b) Are the events independent? [2 marks]

0.8 differs from $P(A) = 0.7$.

Answer: No.

13. In a survey,18 of 30 regular walkers and 22 of 40 other students join a sports club. Compare club membership rates. [3 marks]

$\frac{18}{30} = 0.6; \frac{22}{40} = 0.55.$

The walker rate is 5 percentage points higher.

Answer: 0.6 for walkers and 0.55 for others.

14. A survey finds that 18 of 30 regular walkers and 22 of 40 other students join a sports club. Does this prove that walking causes club membership? Explain one limitation. [3 marks]

The groups may differ in other ways, such as existing interest in sport.

An observational association does not isolate a cause.

Answer: No.

15. Assume rain on two days is independent and each day has rain probability 0.3. Find $P(\text{rain on at least one day})$. [3 marks]

No rain on either day has probability $0.7 \times 0.7 = 0.49$.

The complement is $1 - 0.49 = 0.51$.

Answer: 0.51.

16. Design and describe a simulation to estimate $P(\text{two red counters})$ from 3 red and 2 blue without replacement. State what to record and how to form the estimate. [4 marks]

The shuffle models equal chances for each counter.

No item is reused inside a trial.

Restoring between trials keeps experiments comparable.

Repeat many times and report relative frequency with random variation.

Answer: Randomly shuffle five labelled counters, take the first two, record whether both are red, then restore and reshuffle for each trial. Estimate by RR count divided by trials.