Probability - Practice test 2

16 questions | 45 marks | 60 minutes

Compare models, tables and dependent events.

Equivalent correct methods earn the corresponding marks.

1. A spinner has $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{6}$. Find $P(\text{not } C)$. [2 marks]

$1-\frac{1}{6}=\frac{5}{6}$.

Answer: $\frac{5}{6}$.

2. A spinner has $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{6}$. It is spun twice independently. Find $P(\text{the same letter twice})$. [3 marks]

$P(\text{AA})+P(\text{BB})+P(\text{CC})=\frac{1}{4}+\frac{1}{9}+\frac{1}{36}$.

$\frac{9+4+1}{36}=\frac{14}{36}=\frac{7}{18}$.

Answer: $\frac{7}{18}$.

3. A cafe offers 2 vegetarian mains and 3 other mains, each with 3 drinks. How many vegetarian-main-and-drink choices are there? [2 marks]

Restrict the mains first, then $2\times 3=6$.

Answer: 6.

4. Using digits 1,2,3,4 once each, how many four-digit numbers are even? [3 marks]

Choose the last digit in 2 ways.

Arrange the other three in $3!=6$ ways;$2\times 6=12$.

Answer: 12.

5. How many distinct arrangements of AABC are there? Explain the overcount. [3 marks]

Four labelled letters would give $4! = 24$.

The two identical A letters cause a factor 2 overcount; $\frac{24}{2} = 12$.

Answer: 12.

6. Seven students form an unordered pair. How many pairs? [3 marks]

$7 \times 6 = 42$ ordered selections.

Each pair appears twice, so $\frac{42}{2} = 21$.

Answer: 21.

7. A number is chosen uniformly from 1-10. Given that it is even, find the probability it is greater than 5. [2 marks]

Restrict to $\{2, 4, 6, 8, 10\}$; 6,8,10 satisfy the event.

Answer: $\frac{3}{5}$.

8. Using the club table, find $P(\text{not music} \mid \text{drama})$. [2 marks]

Invented membership data for 60 students.

Music membership	Drama	Not drama	Total
Music	8	12	20
Not music	10	30	40
Total	18	42	60

10 of 18 drama students do not do music; $\frac{10}{18} = \frac{5}{9}$.

Answer: $\frac{5}{9}$.

9. A group has 30 girls and 20 boys. Nine girls and six boys wear glasses. [4 marks]

(a) Find $P(\text{girl} \mid \text{glasses})$. [2 marks]

$\frac{9}{9+6} = \frac{9}{15} = \frac{3}{5}$.

Answer: $\frac{3}{5}$.

(b) Are girl and glasses independent in this data? [2 marks]

$P(\text{girl}) = \frac{30}{50} = \frac{3}{5}$, equal to the conditional probability.

Answer: Yes.

10. A bag has 4 green and 3 yellow counters. Draw two without replacement. Find $P(\text{one of each colour})$. [3 marks]

$\text{GY} = (\frac{4}{7})(\frac{3}{6}) = \frac{2}{7}$.

$\text{YG} = (\frac{3}{7})(\frac{4}{6}) = \frac{2}{7}$;total $\frac{4}{7}$.

Answer: $\frac{4}{7}$.

11. A bag contains 4 green and 3 yellow counters. Draw twice at random without replacement. Find $P(\text{second green} \mid \text{first yellow})$. [2 marks]

Removing yellow leaves 4 green among 6 counters; $\frac{4}{6} = \frac{2}{3}$.

Answer: $\frac{2}{3}$.

12. $P(A) = 0.6$, $P(B) = 0.4$, $P(A \mid B) = 0.75$. [4 marks]

(a) Find $P(A \cap B)$. [2 marks]

$0.75 \times 0.4 = 0.3$.

Answer: 0.3.

(b) Find $P(A \cup B)$. [2 marks]

$0.6 + 0.4 - 0.3 = 0.7$.

Answer: 0.7.

13. A tree has first-stage $P(A) = 0.7$ and $P(\text{not } A) = 0.3$. Given A, $P(B) = 0.2$; given not A, $P(B) = 0.6$. Find $P(B)$. [3 marks]

Add disjoint paths: $(0.7)(0.2) + (0.3)(0.6) = 0.14 + 0.18$.

Answer: 0.32.

14. A simulation records 95 events in 400 trials. Estimate the probability. [2 marks]

$\frac{95}{400} = 0.2375$.

Answer: 0.2375.

15. A fair coin has landed head five times. Under the independent-toss model, find the next-tail probability and explain. [3 marks]

Earlier results do not change the next independent toss.

Answer: $\frac{1}{2}$.

16. In an invented scanner trial,20 unreadable labels are flagged and 5 are not. Of 975 readable labels,80 are flagged. [4 marks]

(a) Find $P(\text{unreadable} \mid \text{flagged})$. [2 marks]

Flagged total $20 + 80 = 100$; $\frac{20}{100} = \frac{1}{5}$.

Answer: $\frac{1}{5}$.

(b) Find $P(\text{flagged} \mid \text{unreadable})$. [2 marks]

Unreadable total 25; $\frac{20}{25} = \frac{4}{5}$.

Answer: $\frac{4}{5}$.