Pythagoras and Trigonometry - Applications test

16 questions | 58 marks | 60 minutes

A more demanding paper combining surveying, design, shadows, bearings, periodic models and checks on assumptions.

Equivalent correct methods earn the corresponding marks.

1. A rectangular practice court is 28 m by 21 m. Find its diagonal. [2 marks]

$d = \sqrt{28^2 + 21^2} = \sqrt{1225} = 35$ m.

Answer: 35 m

2. At one time of day, a 1.8 m vertical post casts a 2.4 m shadow on level ground. [4 marks]

(a) Find the Sun elevation to 0.1°. [2 marks]

$\theta = \tan^{-1}\left(\frac{1.8}{2.4}\right)$.

Answer: 36.9°

(b) At the same time, a vertical tree has an 18 m shadow on the same level ground. Find its height. [2 marks]

The triangles are similar.

$\frac{h}{18} = \frac{1.8}{2.4}$, so $h = 13.5$ m.

Answer: 13.5 m

3. A drone travels 200 m horizontally on bearing 135°. Find its eastward and southward displacements exactly. [3 marks]

The direction is 45° east of south.

Each component is $200\left(\frac{\sqrt{2}}{2}\right) = 100\sqrt{2}$ m.

Answer: $100\sqrt{2}$ m east and $100\sqrt{2}$ m south.

4. A 15 m ladder initially has its foot 9 m from a wall. The foot moves to 12 m from the wall. Find the drop of the ladder top. [3 marks]

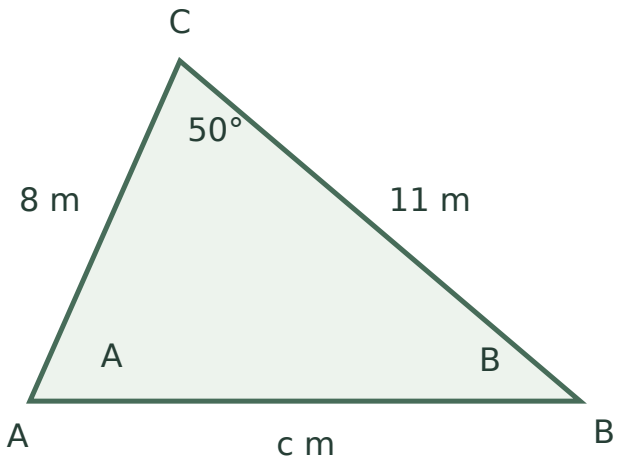
Initial height $= \sqrt{225 - 81} = 12$ m.

Final height $= \sqrt{225 - 144} = 9$ m.

Drop $= 12 - 9 = 3$ m.

Answer: 3 m

5. Two edges of a triangular sail are 8 m and 11 m. Their included angle is 50°. [5 marks]



Sketch not to scale

(a) Find the sail area to 0.1 m². [2 marks]

$A = \frac{1}{2}(8)(11)\sin 50^\circ$.

Answer: 33.7 m²

(b) Find the third edge to 0.1 m. [3 marks]

$c = \sqrt{8^2 + 11^2 - 2(8)(11)\cos 50^\circ}$.

Answer: 8.5 m

6. A survey triangle has angles 47° and 68°. The side opposite 68° is 23 m. Find the side opposite the third angle, to 0.1 m. [3 marks]

Third angle $= 180^\circ - 47^\circ - 68^\circ = 65^\circ$.

$c = \frac{23\sin 65^\circ}{\sin 68^\circ}$.

Answer: 22.5 m

7. A triangular roof brace has lengths 10 cm, 13 cm and 17 cm. Find its largest angle to 0.1° and classify it. [3 marks]

The largest angle is opposite the longest side, 17 cm.

$\cos C = \frac{10^2 + 13^2 - 17^2}{2 \cdot 10 \cdot 13} = -\frac{1}{13}$.

A negative cosine corresponds to an obtuse angle.

Answer: 94.4°; obtuse.

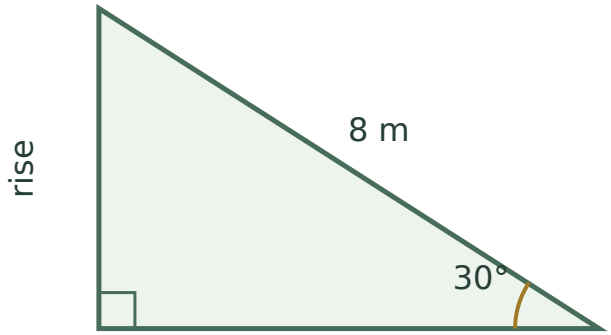
8. Two sides of a triangular garden are fixed at 12 m and 20 m. What is the greatest possible area, and which included angle gives it? [3 marks]

$A = \frac{1}{2}(12)(20)\sin C = 120\sin C$.

The largest possible sine is 1, at 90°.

Answer: 120 m² at 90°.

9. A ramp rises at 30° to level ground and its sloping length is 8 m. [4 marks]



Sketch not to scale

(a) Find its rise exactly. [2 marks]

$h = 8\sin 30^\circ = 4$ m.

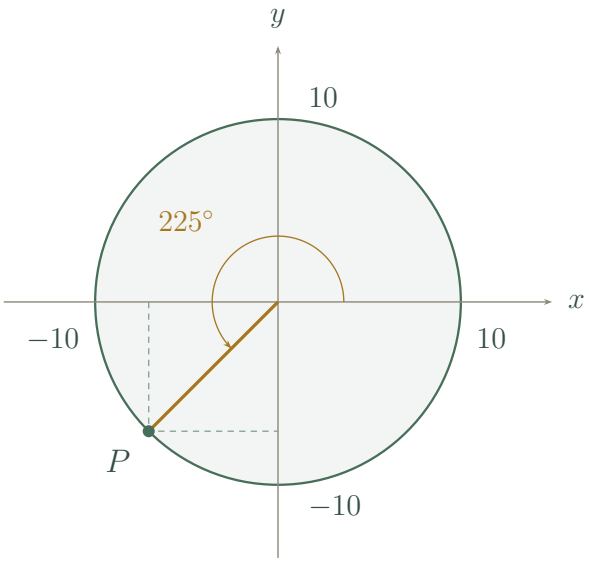
Answer: 4 m

(b) Find its horizontal run exactly. [2 marks]

$d = 8\cos 30^\circ = 4\sqrt{3}$ m.

Answer: $4\sqrt{3}$ m

10. A point lies 10 units from the origin at angle 225° anticlockwise from positive x. Give its exact coordinates. [3 marks]



The unit-circle coordinates are both $-\frac{\sqrt{2}}{2}$.

Multiply each coordinate by 10.

Answer: $(-5\sqrt{2}, -5\sqrt{2})$

11. A model of tide height is $H = 3 + 1.2\cos(30t)$, where H is in metres, t is in hours and the angle is in degrees. [4 marks]

(a) State its period and range. [2 marks]

Period $= \frac{360}{30} = 12$ hours.

Midline is 3 and amplitude 1.2.

Answer: Period 12 hours; $1.8 \le H \le 4.2$ metres.

(b) Find the first time at or after $t = 0$ of maximum height and of minimum height. [2 marks]

Cosine is 1 at 0° and -1 at 180°.

Thus $30t = 0^\circ$ or 180°, respectively.

Answer: Maximum at 0 hours; minimum at 6 hours.

12. For $H = 3 + 1.2\cos(30t)$, find every time between 0 and 12 hours inclusive when $H = 3$ m. Angles are in degrees. [3 marks]

$3 + 1.2\cos(30t) = 3$, so $\cos(30t) = 0$.

Between 0° and 360°, the inside angles are 90° and 270°.

Divide by 30.

Answer: 3 hours and 9 hours.

13. A survey triangle has $A = 30^\circ$, $a = 20$ m and $b = 30$ m. [4 marks]

(a) Find all possible B to 0.1°. [3 marks]

$\sin B = \frac{30\sin 30^\circ}{20} = 0.75$.

The inverse-sine result and its supplement both leave a positive third angle.

Answer: 48.6° or 131.4°

(b) Would the extra statement that B is acute select a unique value? Explain. [1 mark]

The supplementary value is obtuse, so only the acute candidate satisfies the condition.

Answer: Yes: $B = 48.6^\circ$.

14. An estimated height uses $h = 100\tan\theta$ metres. Compare the estimates for $\theta = 10.0^\circ$ and $\theta = 10.1^\circ$. Give each height and the difference to 0.1 m. [4 marks]

$h_1 = 100\tan 10^\circ$ and $h_2 = 100\tan 10.1^\circ$.

Keep both unrounded values before subtracting.

A small measurement change can affect the reported length.

Answer: 17.6 m and 17.8 m; difference 0.2 m.

15. Two points on opposite sides of a vertical tower are 30 m apart on level ground. The tower lies between them. Elevations to its top are 52° and 38°. Find tower height to 0.1 m. [5 marks]

Let the horizontal distances be x and y. Then $x + y = 30$.

$x = \frac{h}{\tan 52^\circ}$ and $y = \frac{h}{\tan 38^\circ}$.

$h(\frac{1}{\tan 52^\circ} + \frac{1}{\tan 38^\circ}) = 30$.

Solve for h, then round.

Answer: 14.6 m

16. A triangular paddock has sides 40 m, 60 m and 75 m. [5 marks]

(a) Find the angle between the 40 m and 60 m sides to 0.1°. [3 marks]

$\cos C = \frac{40^2 + 60^2 - 75^2}{2 \cdot 40 \cdot 60}$.

Use inverse cosine.

Answer: 95.1°

(b) Find the paddock area to 0.1 m². [2 marks]

$A = \frac{1}{2}(40)(60)\sin C$.

Use the unrounded angle or its full-precision sine.

Answer: 1195.3 m²