Exponential relationships - applications test

16 questions | 53 marks | 60 minutes

Apply exact arithmetic, models and logarithmic reasoning to varied contexts with all data supplied.

Equivalent correct methods earn the corresponding marks.

1. An image is enlarged by factor 2 three times. Write one power for the overall factor and evaluate. [2 marks]

Multiply $2 \times 2 \times 2$.

Answer: $2^3 = 8$

2. A cube has volume 216 cm^3. Write its side as a fractional power and evaluate. [2 marks]

The side cubed is the volume.

Answer: $216^{\frac{1}{3}} = 6 \text{ cm}$

3. A square tile has area 72 cm^2. Find its exact side and perimeter. [3 marks]

$\sqrt{72} = 6\sqrt{2}$; perimeter is four sides.

Answer: $6\sqrt{2} \text{ cm}$ and $24\sqrt{2} \text{ cm}$

4. A rectangle has length $\sqrt{12} + \sqrt{3}$ cm and width $\sqrt{3}$ cm. Find its exact area. [3 marks]

Length is $3\sqrt{3}$ cm; area is $3\sqrt{3} \times \sqrt{3} = 9$.

Answer: 9 cm^2

5. A path is $\frac{10}{\sqrt{2}}$ m long. Rationalise and round to 2 decimal places. [3 marks]

$\frac{10\sqrt{2}}{2} = 5\sqrt{2} = 7.0710\ldots$

Answer: $5\sqrt{2} \text{ m} \approx 7.07 \text{ m}$

6. The table describes two models at equal time steps. [4 marks]

Time	0	1	2	3
A	8	14	20	26
B	10	20	40	80

(a) Identify the linear and exponential model. [2 marks]

A adds 6 each step; B doubles.

Answer: A is linear; B is exponential.

(b) Give the next value in each model. [2 marks]

$26+6=32$ and $80\times 2=160$.

Answer: A: 32; B: 160.

7. A data-storage model is $D=30(1.25)^t$ GB after t years. Find the value at year 2 and the annual percentage increase. [3 marks]

$30(1.25)^2=46.875$; the factor exceeds 1 by 0.25.

Answer: 46.875 GB; 25% increase.

8. A filtering process retains 80% of a substance each pass from an initial 500 mg. Find the amount after 4 passes. [3 marks]

$500(0.8)^4=204.8$.

Answer: 204.8 mg

9. A model doubles every 3 days from 12 units. Write a formula in days t and find the value at day 9. [3 marks]

At day 9 there are $\frac{9}{3}=3$ doubling periods, giving $12\times 8=96$.

Answer: $Q=12(2^{\frac{t}{3}})$; 96 units.

10. A cooling model is $T = 18 + 32(0.5)^t$ degrees Celsius, with t in hours. [6 marks]

(a) Find the initial temperature. [2 marks]

At zero, $T = 18 + 32 = 50$.

Answer: 50°C

(b) Find when the model reaches 22°C. [2 marks]

$22 - 18 = 32(0.5)^t$ gives $(0.5)^t = \frac{1}{8} = (0.5)^3$.

Answer: 3 hours

(c) Interpret its horizontal asymptote. [2 marks]

The exponential part approaches zero and the fixed term remains.

Answer: The temperature approaches 18°C.

11. Under $M = \log_{10}(\frac{A}{A_0})$, signal X has $M = 3.2$ and signal Y has $M = 5.2$. How many times larger is Y in amplitude? [3 marks]

$10^{5.2-3.2} = 10^2 = 100$.

Answer: 100 times

12. A concentration model uses $p = -\log_{10} c$. The concentration changes from 10^{-3} to 10^{-5}. Find the change in p and the concentration factor. [3 marks]

The scale values are 3 and 5; the concentration ratio is $\frac{10^{-5}}{10^{-3}} = 10^{-2}$.

Answer: p increases by 2; concentration factor $\frac{1}{100}$.

13. Solve $6(1.3)^t = 15$ to 2 decimal places. [3 marks]

$t = \frac{\log(2.5)}{\log(1.3)} = 3.492437\ldots$.

Answer: $t \approx 3.49$

14. For $y = 2^{x-2} + 1$, state its asymptote and the point when $x = 2$. [2 marks]

The shift gives $y = 1$; at $x = 2$, $2^0 + 1 = 2$.

Answer: $y = 1$ and $(2, 2)$.

15. A model is $N = 200(0.9)^t$ with monthly measurements. [6 marks]

(a) Predict at month 2. [2 marks]

$200(0.9)^2 = 162$.

Answer: 162

(b) The observation is 170. Find the residual. [2 marks]

Observed minus predicted is $170 - 162 = 8$.

Answer: 8

(c) Does one nonzero residual prove the model is useless? Explain. [2 marks]

A discrepancy may reflect observation variation or a changed factor. More observations and a relevant tolerance are needed.

Answer: No; use context and further data.

16. A claim says that fitting an exponential rule to three months of downloads proves the same growth will last ten years. Explain two flaws. [4 marks]

Many models can fit a few points. Audience, capacity and the growth factor can change over ten years.

Answer: A small data set does not uniquely establish the rule; extrapolation assumes unchanged conditions.