Exponential relationships - practice test 2

16 questions | 47 marks | 60 minutes

Original progressive questions on exact surds, indices, graphs and exponential/logarithmic reasoning.

Equivalent correct methods earn the corresponding marks.

1. Simplify $\frac{(p^4)^3}{p^5}$ for $p \neq 0$. [2 marks]

$4 \times 3 - 5 = 7$.

Answer: p^7

2. Evaluate $32^{\frac{3}{5}}$. [2 marks]

The fifth root is 2; cube it.

Answer: 8

3. Write $\sqrt[3]{a^2}$ as a power for $a > 0$. [2 marks]

The denominator names the cube root.

Answer: $a^{\frac{2}{3}}$

4. Simplify $\sqrt{147}$. [2 marks]

$147 = 49 \times 3$.

Answer: $7\sqrt{3}$

5. Simplify $(\sqrt{6} + \sqrt{2})^2$. [3 marks]

$6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3}$.

Answer: $8 + 4\sqrt{3}$

6. Rationalise $\frac{1}{4 - \sqrt{7}}$. [3 marks]

The conjugate product in the denominator is $16 - 7 = 9$.

Answer: $\frac{4 + \sqrt{7}}{9}$

7. A square has area $75\ \text{m}^2$. Find its exact perimeter. [3 marks]

The side is $5\sqrt{3}$ m; multiply by 4.

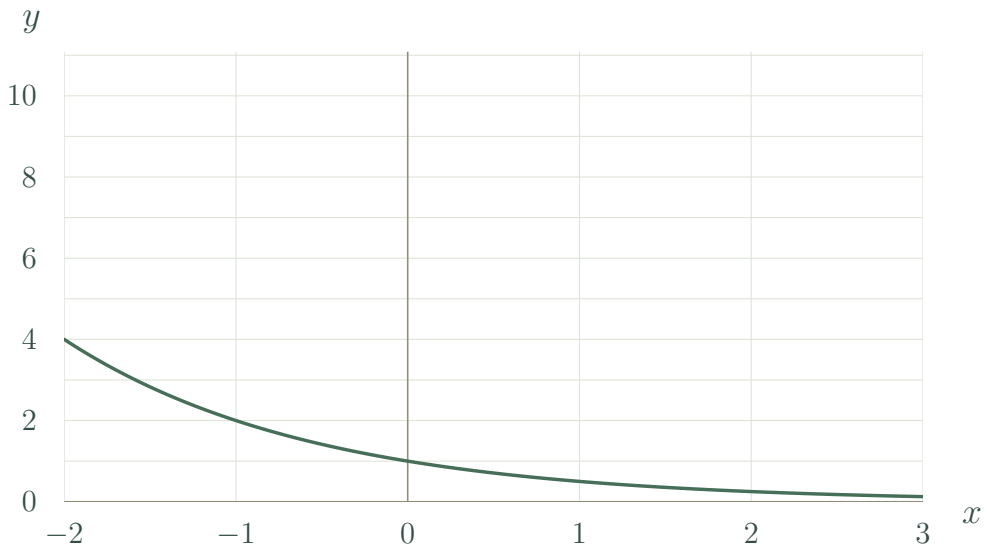
Answer: $20\sqrt{3}\ \text{m}$

8. A student rounds $\sqrt{5}$ to 2.2 before squaring. Find the exact result, the rounded result and their difference. [3 marks]

$(\sqrt{5})^2 = 5$, $2.2^2 = 4.84$, and $5 - 4.84 = 0.16$.

Answer: 5; 4.84; difference 0.16.

9. Consider $y = (\frac{1}{2})^x$. [5 marks]



(a) Find its outputs at $x = -2, -1, 0, 1, 2$. [2 marks]

Negative exponents give reciprocals of powers.

Answer: $4, 2, 1, \frac{1}{2}, \frac{1}{4}$

(b) Sketch it on the grid and state its range. [3 marks]

Draw a smooth decreasing positive curve through the points, approaching $y = 0$.

Answer: $y > 0$

10. A sample is 160 g and retains 75% of its current mass each hour. Find its mass after 3 hours. [3 marks]

$160(0.75)^3 = 67.5$ g.

Answer: 67.5 g

11. Outputs $3, 8, 15, 24$ correspond to inputs $0, 1, 2, 3$. Use differences to identify a suitable model family. [3 marks]

First differences $5, 7, 9$ give constant second difference 2.

Answer: Quadratic candidate

12. Solve $2^{x+1} = 11$ to 3 decimal places. [3 marks]

$x = \frac{\log 11}{\log 2} - 1 = 2.459431\ldots$.

Answer: $x \approx 2.459$

13. Find the domain of $\log_5(2x - 6)$. [2 marks]

Require $2x - 6 > 0$.

Answer: $x > 3$

14. Solve $\log_2 x + \log_2(x - 2) = 3$. [4 marks]

Domain $x > 2$; $x(x - 2) = 8$, so $(x - 4)(x + 2) = 0$ and only 4 is valid.

Answer: $x = 4$

15. Two quantities have base-10 log-scale readings 2 and 5. Find their larger-to-smaller ratio. [2 marks]

$10^{5-2} = 1000$.

Answer: 1000

16. A graph has equation $y = a(3^x) + 2$. [5 marks]

(a) It passes through $(0, 7)$. Find a. [2 marks]

$7 = a + 2$, so $a = 5$.

Answer: $a = 5$

(b) Find its value at $x = 2$ and explain why consecutive output ratios are not always 3. [3 marks]

$5(9) + 2 = 47$. Distances above $y = 2$ multiply by 3, but the actual outputs have an added constant.

Answer: 47; the added 2 changes output ratios.