Quadratics and Parabolas - Practice test 2

16 questions | 51 marks | 55 minutes

A second balanced paper using construction, error analysis, intersections and quadratic applications.

Equivalent correct methods earn the corresponding marks.

1. A student writes $(x-3)^2 = x^2 + 9$. Correct the expansion and identify the missing term. [2 marks]

The two cross-products are each $-3x$.

Answer: $x^2 - 6x + 9$; missing term $-6x$.

2. Write a monic quadratic expression with roots -2 and 5, then expand it. [3 marks]

The root factors are $x+2$ and $x-5$.

Answer: $(x+2)(x-5) = x^2 - 3x - 10$

3. Factorise $4x^2 - 25$. [2 marks]

Use the difference of squares $(2x)^2 - 5^2$.

Answer: $(2x-5)(2x+5)$

4. Solve $4x^2 - 25 = 0$. [2 marks]

$4x^2 = 25$, so $x^2 = \frac{25}{4}$.

Answer: $x = \pm\frac{5}{2}$

5. Solve $2(x+1)(x-3)=16$ exactly. [4 marks]

$x^2-2x-3=8$, so $(x-1)^2=12$.

Answer: $x=1\pm 2\sqrt{3}$

6. Write $y=-2x^2+8x+1$ in vertex form and state its greatest value. [3 marks]

$-2(x^2-4x)+1=-2[(x-2)^2-4]+1$.

Answer: $y=-2(x-2)^2+9$; greatest value 9.

7. Use the quadratic formula to solve $3x^2-2x-5=0$. [3 marks]

$x=\frac{2\pm\sqrt{4+60}}{6}=\frac{2\pm 8}{6}$.

Answer: $x=\frac{5}{3}$ or $x=-1$

8. Consider $x^2+kx+4=0$, where k is real. [4 marks]

(a) State the k-values giving two distinct real roots. [2 marks]

The discriminant is k^2-16; require $k^2>16$.

Answer: $k<-4$ or $k>4$.

(b) State the k-values giving one repeated real root. [1 mark]

Require $k^2-16=0$.

Answer: $k=\pm 4$.

(c) State the k-values giving no real roots. [1 mark]

Require $k^2-16<0$.

Answer: $-4<k<4$.

9. A parabola has vertex $(-1, -8)$ and passes through $(0, -6)$. Find its rule and its roots. [4 marks]

$-6 = a(1)^2 - 8$ gives $a = 2$.

For roots, $(x+1)^2 = 4$.

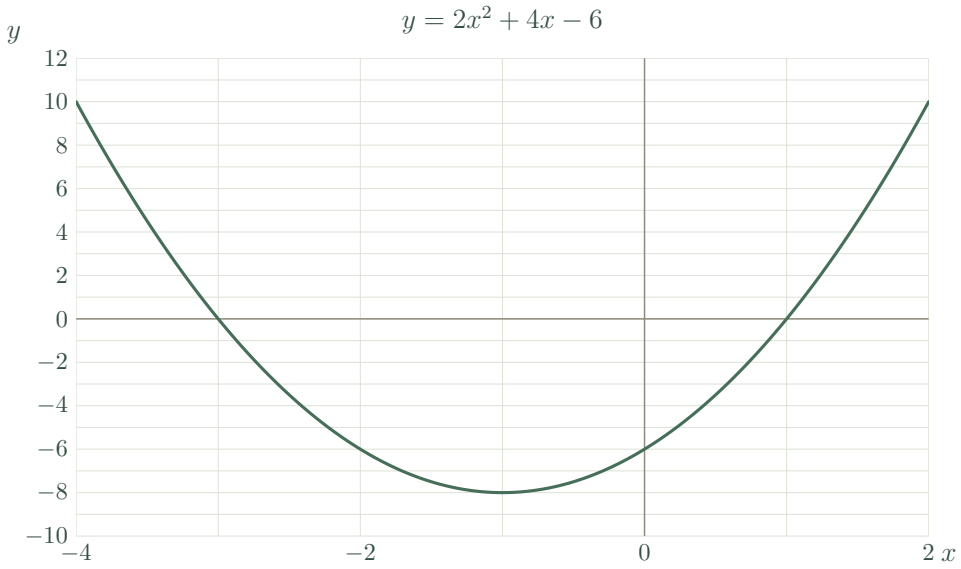
Answer: $y = 2(x+1)^2 - 8$; roots $-3, 1$.

10. The rule is $y = -x^2 + 6x - 5$. Find its roots, vertex and y-intercept. [4 marks]

$y = -(x-1)(x-5) = -(x-3)^2 + 4$.

Answer: Roots $1, 5$; vertex $(3, 4)$; y-intercept $(0, -5)$.

11. Sketch $y = 2x^2 + 4x - 6$. Label the vertex and all intercepts. [4 marks]



$y = 2(x+3)(x-1) = 2(x+1)^2 - 8$.

Answer: Vertex $(-1, -8)$; roots $-3, 1$; y-intercept $(0, -6)$.

12. Use the table to locate a positive root of $x^2 + x - 4 = 0$ between two listed values. [3 marks]

x	1.56	1.57
$x^2 + x - 4$	-0.0064	0.0349

The continuous polynomial has opposite signs at the two listed inputs.

Answer: $1.56 < x < 1.57$

13. Find all intersections of $y = x^2 + 1$ and $y = 2x$. Explain how many distinct intersections there are. [3 marks]

$x^2 - 2x + 1 = (x - 1)^2 = 0$ has the repeated root 1.

Answer: One intersection, $(1, 2)$.

14. A triangle has height x cm and base $(x + 3)$ cm. Its area is 20 cm². Find the height and base. [4 marks]

$\frac{1}{2}x(x + 3) = 20$ gives $x^2 + 3x - 40 = (x + 8)(x - 5) = 0$.

Use the positive height 5.

Answer: Height 5 cm; base 8 cm.

15. The function $y = 2x^2 + kx - 12$ has a root at $x = 3$. Find k and the other root. [4 marks]

$18 + 3k - 12 = 0$, so $k = -2$.

$2x^2 - 2x - 12 = 2(x - 3)(x + 2)$.

Answer: $k = -2$; other root -2.

16. A student says the vertex of $y = (x - 4)^2 + 7$ is $(-4, 7)$. Explain and correct the mistake. [2 marks]

The squared expression is zero when $x - 4 = 0$, which gives $x = 4$.

Answer: The vertex is $(4, 7)$.