Quadratic applications and model checks

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. A rectangle is 3 cm longer than it is wide and has area 54 cm². Find its dimensions. [3 marks]

$x(x+3)=54$, so $(x+9)(x-6)=0$.

The positive width is 6.

Answer: 6 cm by 9 cm.

2. A photo is 6 cm by 10 cm. A uniform border makes the total area 96 cm². Find the border width. [4 marks]

$(6+2x)(10+2x)=96$.

$4x^2+32x-36=0$, or $(x+9)(x-1)=0$.

The positive border width is 1.

Answer: 1 cm.

3. A ball's height is $h=-5t^2+20t+1$ metres after t seconds. Find its maximum height and when it occurs. [3 marks]

$h=-5(t-2)^2+21$.

Answer: Maximum 21 m at $t=2$ s.

4. A model gives ticket revenue $R = p(80 - 2p)$ dollars when price p dollars satisfies $0 \le p \le 40$. Find the price giving greatest predicted revenue. [3 marks]

$R = -2(p - 20)^2 + 800.$

The allowed price $p = 20$ gives 40 tickets.

Answer: Price 20; revenue 800.

5. A design width x metres must satisfy $x^2 - 5x + 6 = 0$ and $x > 2$. Which root is allowed? [2 marks]

$(x - 2)(x - 3) = 0$ gives $2, 3.$

The strict condition excludes 2.

Answer: $x = 3$ m.

6. A proposed height model is $h = -t^2 + 6t + 2$. At $t = 4$, a measurement is 9.5 m. Find the model's error there and explain whether one check proves the model valid at all times. [4 marks]

$h(4) = -16 + 24 + 2 = 10.$

$10 - 9.5 = 0.5.$

Test more values in the intended interval and consider the physical assumptions.

Answer: Prediction 10 m; overestimate 0.5 m. One check does not establish validity for all times.