Graphical/numerical solutions

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Use the table to locate the positive root of $x^2 - 3 = 0$ between two listed values. [2 marks]

x	1.6	1.7	1.8
$x^2 - 3$	-0.44	-0.11	0.24

$f(1.7) = -0.11$ and $f(1.8) = 0.24$ have opposite signs.

Answer: $1.7 < x < 1.8$

2. Estimate both roots of $x^2 - 5 = 0$ to two decimal places. [2 marks]

The exact roots are $\pm\sqrt{5} \approx \pm 2.236068$.

Answer: $x \approx -2.24,\ 2.24$

3. Find the intersections of $y = x^2 - 2$ and $y = x$. [3 marks]

$x^2 - x - 2 = (x - 2)(x + 1) = 0$; then $y = x$.

Answer: $(-1, -1)$ and $(2, 2)$

4. Use the supplied values to round the positive root of $x^2 + x - 1 = 0$ to two decimal places. [3 marks]

x	0.618	0.619
$x^2 + x - 1$	-0.000076	0.002161

The root is between 0.618 and 0.619, so it rounds to 0.62.

Answer: $x \approx 0.62$

5. Solve $x^2 = 2x + 1$ exactly, then give decimal answers to two places. [3 marks]

Complete the square: $(x-1)^2 = 2$.

Answer: $x = 1 \pm \sqrt{2} \approx -0.41,\ 2.41$

6. Why does a sign-change search miss the root of $f(x) = (x-2)^2$ if it checks only $x = 1$ and $x = 3$? [3 marks]

$f(1) = f(3) = 1$, but the curve touches the axis at its repeated root 2.

Answer: Both endpoint values are positive even though $f(2) = 0$.