Completing the square

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Complete the square: $x^2 + 6x + 5$. [2 marks]

$x^2 + 6x + 9 - 9 + 5 = (x + 3)^2 - 4$.

Answer: $(x + 3)^2 - 4$

2. Complete the square: $x^2 - 10x + 8$. [2 marks]

$x^2 - 10x + 25 - 25 + 8 = (x - 5)^2 - 17$.

Answer: $(x - 5)^2 - 17$

3. Complete the square: $3x^2 + 12x + 1$. [3 marks]

$3(x^2 + 4x) + 1 = 3[(x + 2)^2 - 4] + 1$.

Answer: $3(x + 2)^2 - 11$

4. Solve $x^2 + 2x - 4 = 0$ by completing the square. [3 marks]

$(x + 1)^2 = 5$, so $x + 1 = \pm\sqrt{5}$.

Answer: $x = -1 \pm \sqrt{5}$

5. Write $x^2 + 3x + 1$ in completed-square form. [3 marks]

Add and subtract $\frac{9}{4}$: $(x + \frac{3}{2})^2 + 1 - \frac{9}{4}$.

Answer: $\left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$

6. Find the minimum value of $2x^2 + 8x + 1$ and the x-value where it occurs. [3 marks]

$2x^2 + 8x + 1 = 2(x + 2)^2 - 7$.

The squared term is zero at $x = -2$ and never negative.

Answer: Minimum -7 at $x = -2$.