Sampling and confidence intervals - Paper 6

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Combine model checks, inverse calculations and applied evaluation in a demanding review. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A researcher must sample 80 workers from shifts with populations 250, 150 and 100. [6 marks]

(a) Give proportional allocations and describe the random step. [3 marks]

(b) A manager supplies only volunteers from each shift. Does retaining those same allocations remove self-selection bias? [3 marks]

2. A right-skewed finite-variance population has mean 60 and SD 15. An independent sample has $n = 225$. [6 marks]

(a) Find the centre and standard error of the sample mean and state the normality status. [3 marks]

(b) Use that model to approximate the probability that the mean is between 58.5 and 61.5. [3 marks]

3. A mean estimate is 75.6 and the known-spread standard error is 0.8. [6 marks]

(a) Find the 95% and 99% intervals. [3 marks]

(b) A target of 77.3 lies outside one interval but inside the other. Explain without contradiction. [3 marks]

4. A random representative sample of 800 people has 432 successes. [6 marks]

(a) Find the estimate and 95% proportion interval. [3 marks]

(b) Assess a claim of more than half the population under the interval model. [3 marks]

5. A data collector expects 60% of contacted residents to give a valid response. It needs a conservative 95% proportion margin at most 0.035. [6 marks]

(a) Find the required number of valid responses. [3 marks]

(b) Find a contact-budget estimate and explain one reason it is not a guarantee. [3 marks]

6. A mean study has known population SD 10 and $n = 100$. A new population has SD 15. [6 marks]

(a) Find the sample size needed in the new population to keep standard error unchanged. [3 marks]

(b) If confidence also rises from 95% to 99%, will the same n keep interval width unchanged? Explain. [3 marks]

7. A trial records one success in 40 independent observations. [6 marks]

(a) Calculate the estimate and explain why the usual symmetric interval is questionable. [3 marks]

(b) Would a zero-success follow-up prove the population probability is zero? [3 marks]

8. A claimed 95% procedure is applied repeatedly under its own valid assumptions. [6 marks]

(a) In 2,000 independent repetitions, give expected misses and explain the role of random variation. [3 marks]

(b) A reader says each realised interval has a fixed parameter that is “95% random”. Correct the language. [3 marks]

9. A report compares two treatments using separate intervals: A $(20.1, 21.3)$, B $(20.7, 21.9)$. [6 marks]

(a) Find the estimates and margins. [3 marks]

(b) Explain why overlap alone establishes neither equality nor a formal conclusion about the difference. [3 marks]

10. A large, carefully randomised 2026 sample estimates average screen time with a narrow interval. Measurements come from self-reported recollections. [6 marks]

(a) Name a remaining uncertainty not captured by the sampling margin. [3 marks]

(b) Give a responsible reporting sentence and one possible validation improvement. [3 marks]