Sampling and confidence intervals - Paper 5

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Work backwards from intervals and compare precision under different designs. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A report gives a 95% mean confidence interval $(18.04, 21.96)$, based on $n = 49$ and a known population SD. [6 marks]

(a) Recover the estimate, margin and standard error. [3 marks]

(b) Recover the population SD. [3 marks]

2. A random sample of 400 gives an estimated support proportion 0.55. [6 marks]

(a) Find a 98% normal-approximation interval. [3 marks]

(b) Explain why the same sample's 90% interval is narrower. [3 marks]

3. Study A samples 64 observations from a population with SD 12. Study B samples 144 observations from a population with SD 15. [6 marks]

(a) Find both standard errors. [3 marks]

(b) Which has the narrower interval at a common confidence level, and why is population spread alone insufficient? [3 marks]

4. For a mean with known SD 20, a 95% margin no larger than 1.5 is required. [6 marks]

(a) Find the minimum valid sample size. [3 marks]

(b) A student chooses 682 because the decimal part “does not matter”. Explain the error. [3 marks]

5. A planner uses $p_0 = 0.2$ for a 95% proportion margin 0.03. [6 marks]

(a) Find the planned sample size. [3 marks]

(b) Compare it with the no-prior-estimate conservative plan and explain the risk. [3 marks]

6. An interval claims “95% confidence” but uses the normal critical value 1.645 in a symmetric two-sided calculation. [6 marks]

(a) What two-sided confidence level does that value approximately correspond to? [3 marks]

(b) How should the 95% calculation change with the same data? [3 marks]

7. Researchers use a random sample of customers who made a purchase to estimate satisfaction among all visitors, including those who left without buying. [6 marks]

(a) Identify the mismatch. [3 marks]

(b) Suggest a better frame and a measurement safeguard. [3 marks]

8. Assume $p = 0.45$ and $n = 500$ independent binary observations. [6 marks]

(a) Check the expected success/failure counts. [3 marks]

(b) Approximate $P(\hat{p} > 0.50)$ without continuity correction. [3 marks]

9. A study reports an estimate 0.38 and a 95% interval margin of 0.04. It wishes to halve the margin. [6 marks]

(a) State the approximate sample-size factor if the planning proportion is held fixed. [3 marks]

(b) Why need the actual margin from a new sample not be exactly half? [3 marks]

10. An analyst finds an interval excluding a policy target by only 0.001, but the survey uses leading wording. [6 marks]

(a) What does the small numerical exclusion fail to address? [3 marks]

(b) What should a responsible conclusion include? [3 marks]