Sampling and confidence intervals - Paper 5

10 questions | 60 marks | 75 minutes

Work backwards from intervals and compare precision under different designs. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Equivalent correct methods earn the corresponding marks.

1. A report gives a 95% mean confidence interval $(18.04, 21.96)$, based on $n = 49$ and a known population SD. [6 marks]

(a) Recover the estimate, margin and standard error. [3 marks]

Use midpoint, half-width and $\frac{\text{margin}}{1.96}$.

Answer: Estimate 20; margin 1.96; SE 1.

(b) Recover the population SD. [3 marks]

Multiply SE by $\sqrt{49}$.

Answer: 7.

2. A random sample of 400 gives an estimated support proportion 0.55. [6 marks]

(a) Find a 98% normal-approximation interval. [3 marks]

Use estimated SE $\sqrt{\frac{0.55 \times 0.45}{400}}$ and critical value 2.326.

Answer: $(0.4921, 0.6079)$

(b) Explain why the same sample's 90% interval is narrower. [3 marks]

The estimate and estimated SE do not change.

Answer: Its smaller critical value gives less long-run coverage but greater precision.

3. Study A samples 64 observations from a population with SD 12. Study B samples 144 observations from a population with SD 15. [6 marks]

(a) Find both standard errors. [3 marks]

$\frac{12}{8} = 1.5$ and $\frac{15}{12} = 1.25$.

Answer: A: 1.5; B: 1.25.

(b) Which has the narrower interval at a common confidence level, and why is population spread alone insufficient? [3 marks]

B's larger individual SD is offset by its larger n.

Answer: B is narrower. Sample size as well as spread determines SE.

4. For a mean with known SD 20, a 95% margin no larger than 1.5 is required. [6 marks]

(a) Find the minimum valid sample size. [3 marks]

$n \geq \left(\frac{1.96 \times 20}{1.5}\right)^2 = 682.9511\ldots$; round up.

Answer: 683

(b) A student chooses 682 because the decimal part "does not matter". Explain the error. [3 marks]

Sample-size planning requires rounding up, not truncation.

Answer: 682 lies below the lower bound; it does not meet the target.

5. A planner uses $p_0 = 0.2$ for a 95% proportion margin 0.03. [6 marks]

(a) Find the planned sample size. [3 marks]

$\frac{1.96^2 \times 0.2 \times 0.8}{0.03^2} = 682.9511\ldots$; round up.

Answer: 683

(b) Compare it with the no-prior-estimate conservative plan and explain the risk. [3 marks]

If the actual p is nearer 0.5, the achieved margin can be larger than planned.

Answer: Conservative plan 1068; the smaller plan relies on a defensible p_0 near 0.2.

6. An interval claims “95% confidence” but uses the normal critical value 1.645 in a symmetric two-sided calculation. [6 marks]

(a) What two-sided confidence level does that value approximately correspond to? [3 marks]

Central probability between -1.645 and 1.645 is approximately 0.90.

Answer: 90%.

(b) How should the 95% calculation change with the same data? [3 marks]

Do not relabel a 90% procedure as 95%.

Answer: Use about 1.96, widening the interval.

7. Researchers use a random sample of customers who made a purchase to estimate satisfaction among all visitors, including those who left without buying. [6 marks]

(a) Identify the mismatch. [3 marks]

Random sampling inside the purchaser frame does not represent all visitors automatically.

Answer: The sampling frame excludes non-purchasing visitors.

(b) Suggest a better frame and a measurement safeguard. [3 marks]

Also monitor response and define the visitor/time target clearly.

Answer: Sample from the visitor population and use neutral consistent satisfaction wording.

8. Assume $p = 0.45$ and $n = 500$ independent binary observations. [6 marks]

(a) Check the expected success/failure counts. [3 marks]

Multiply n by p and $1 - p$.

Answer: 225 and 275.

(b) Approximate $P(\hat{p} > 0.50)$ without continuity correction. [3 marks]

Use mean 0.45 and SD $\sqrt{\frac{0.45 \times 0.55}{500}}$.

Evaluate the upper normal tail.

Answer: 0.0123

9. A study reports an estimate 0.38 and a 95% interval margin of 0.04. It wishes to halve the margin. [6 marks]

(a) State the approximate sample-size factor if the planning proportion is held fixed. [3 marks]

Margin is proportional to $\frac{1}{\sqrt{n}}$.

Answer: Four times the sample size.

(b) Why need the actual margin from a new sample not be exactly half? [3 marks]

The square-root comparison holds with the probability factor fixed.

Answer: The new sample proportion can change, altering the estimated variance factor.

10. An analyst finds an interval excluding a policy target by only 0.001, but the survey uses leading wording. [6 marks]

(a) What does the small numerical exclusion fail to address? [3 marks]

The sampling interval does not quantify that systematic error.

Answer: Response bias from wording.

(b) What should a responsible conclusion include? [3 marks]

Avoid presenting a tiny conditional exclusion as decisive evidence from a biased study.

Answer: The interval and model assumptions, the wording limitation and the practical size of the discrepancy.