Sampling and confidence intervals - Paper 4

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Evaluate uncertainty in production and quality-assurance studies. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A technician chooses ten “typical-looking” items from each of ten batches. [6 marks]

(a) Is this a random sample within batches? Explain. [3 marks]

(b) Give a random within-batch alternative and explain why batches matter. [3 marks]

2. A production population has mean 40 and SD 12. Measurements are independent but strongly right-skewed. [6 marks]

(a) Find the centre and standard error of the mean for $n = 144$. [3 marks]

(b) Explain why the same normal approximation need not be adequate at $n = 4$. [3 marks]

3. A random independent sample of 100 components has mean strength 71.2 MPa. Population SD is known as 8 MPa; a normal mean model is justified. [6 marks]

(a) Find a 99% interval. [3 marks]

(b) What would changing to 95% confidence do to the centre and width? [3 marks]

4. A monitor gives a large sample SD after occasional extreme readings. The analyst removes those readings solely to narrow the interval. [6 marks]

(a) Explain why this is not a defensible automatic rule. [3 marks]

(b) What should be reported when exclusion is justified? [3 marks]

5. In 300 randomly selected independently manufactured components, 18 are defective. [6 marks]

(a) Find the defect estimate and a 95% approximate interval. [3 marks]

(b) Check the observed-count screening rule and state a remaining design issue. [3 marks]

6. A correct 98% interval method is repeated independently 500 times. [6 marks]

(a) Give expected misses and explain whether the count is fixed. [3 marks]

(b) An observed interval misses the true parameter. Does that alone invalidate the method? [3 marks]

7. A 95% mean interval has margin 1.176 units and $n = 100$. [6 marks]

(a) Recover the known population SD used in the calculation. [3 marks]

(b) Find the minimum n for margin at most 0.75 at the same confidence/spread. [3 marks]

8. A trial has 3 successes among 50 independent observations. [6 marks]

(a) Calculate the sample estimate and explain a problem with a symmetric normal interval. [3 marks]

(b) Would truncating a negative endpoint to zero prove the method reliable? [3 marks]

9. A larger sample is proposed to reduce the margin from 1.5 units to 0.5 units. Current n is 80; confidence and population spread are fixed. [6 marks]

(a) Find the required size by the scaling argument. [3 marks]

(b) If the population spread also doubles, what size is required for that same target margin? [3 marks]

10. An interval for mean load is $(98.9, 101.1)$ N. A safety threshold requires the true mean to be at least 100 N. [6 marks]

(a) Does the complete interval establish compliance? [3 marks]

(b) Explain why a statement about a mean alone does not establish safety of every component. [3 marks]