Sampling and confidence intervals - Paper 4

10 questions | 60 marks | 75 minutes

Evaluate uncertainty in production and quality-assurance studies. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Equivalent correct methods earn the corresponding marks.

1. A technician chooses ten “typical-looking” items from each of ten batches. [6 marks]

(a) Is this a random sample within batches? Explain. [3 marks]

Apparently typical items may exclude unusual but important outcomes.

Answer: No. The technician’s judgement determines inclusion.

(b) Give a random within-batch alternative and explain why batches matter. [3 marks]

Batch conditions may create shared effects, so representation and independence need attention.

Answer: Number items within each batch and select labels randomly; sample across relevant batches.

2. A production population has mean 40 and SD 12. Measurements are independent but strongly right-skewed. [6 marks]

(a) Find the centre and standard error of the mean for $n = 144$. [3 marks]

$\frac{12}{\sqrt{144}} = 1.$

Answer: Centre 40; SE 1.

(b) Explain why the same normal approximation need not be adequate at $n = 4$. [3 marks]

The limiting central-limit result is not a universal small-sample rule.

Answer: A mean of only four strongly skewed observations can remain skewed.

3. A random independent sample of 100 components has mean strength 71.2 MPa. Population SD is known as 8 MPa; a normal mean model is justified. [6 marks]

(a) Find a 99% interval. [3 marks]

SE $= 0.8$ MPa; margin $= 2.0608$ MPa.

Answer: $(69.1392, 73.2608)$

(b) What would changing to 95% confidence do to the centre and width? [3 marks]

Only the critical value changes for the same data/model.

Answer: Centre unchanged; width smaller.

4. A monitor gives a large sample SD after occasional extreme readings. The analyst removes those readings solely to narrow the interval. [6 marks]

(a) Explain why this is not a defensible automatic rule. [3 marks]

A reason such as documented measurement error needs evidence and transparent reporting.

Answer: Removing genuine extremes can change the target distribution and bias the result.

(b) What should be reported when exclusion is justified? [3 marks]

Separate a pre-specified quality rule from choosing a favourable result after inspection.

Answer: The rule, reason, number excluded and effect on the conclusion.

5. In 300 randomly selected independently manufactured components, 18 are defective. [6 marks]

(a) Find the defect estimate and a 95% approximate interval. [3 marks]

Estimated SE $= \sqrt{\frac{0.06 \times 0.94}{300}}$.

Use 1.96 times the estimated SE.

Answer: Estimate 0.06; interval $(0.0331, 0.0869)$.

(b) Check the observed-count screening rule and state a remaining design issue. [3 marks]

A numerical count check is not a sampling-design check.

Answer: Counts 18 and 282 pass the usual screening rule; independence/representative batch selection must still hold.

6. A correct 98% interval method is repeated independently 500 times. [6 marks]

(a) Give expected misses and explain whether the count is fixed. [3 marks]

$500 \times 0.02 = 10$.

Answer: 10 expected misses; the count varies randomly.

(b) An observed interval misses the true parameter. Does that alone invalidate the method? [3 marks]

Investigate systematic errors if present, but one miss is compatible with advertised coverage.

Answer: No; occasional misses are expected.

7. A 95% mean interval has margin 1.176 units and $n = 100$. [6 marks]

(a) Recover the known population SD used in the calculation. [3 marks]

SE $= \frac{1.176}{1.96} = 0.6$; multiply by $\sqrt{100}$.

Answer: 6 units.

(b) Find the minimum n for margin at most 0.75 at the same confidence/spread. [3 marks]

$n \geq \left(\frac{1.96 \times 6}{0.75}\right)^2 = 245.8624$; round up.

Answer: 246

8. A trial has 3 successes among 50 independent observations. [6 marks]

(a) Calculate the sample estimate and explain a problem with a symmetric normal interval. [3 marks]

The observed success-count screening rule fails.

Answer: Estimate 0.06; the success count is only 3, so normal/Wald coverage may be poor.

(b) Would truncating a negative endpoint to zero prove the method reliable? [3 marks]

Choose a method appropriate to the count model rather than cosmetically changing endpoints.

Answer: No. Clipping does not restore the advertised coverage.

9. A larger sample is proposed to reduce the margin from 1.5 units to 0.5 units. Current n is 80; confidence and population spread are fixed. [6 marks]

(a) Find the required size by the scaling argument. [3 marks]

A threefold margin reduction needs nine times n.

Answer: 720.

(b) If the population spread also doubles, what size is required for that same target margin? [3 marks]

Spread doubling requires a further factor of four in n.

Answer: 2,880.

10. An interval for mean load is $(98.9, 101.1)$ N. A safety threshold requires the true mean to be at least 100 N. [6 marks]

(a) Does the complete interval establish compliance? [3 marks]

Being centred at 100 does not make the lower bound satisfy the requirement.

Answer: No; it includes means below 100 N.

(b) Explain why a statement about a mean alone does not establish safety of every component. [3 marks]

Assessing an individual-tail failure risk needs more than a mean confidence interval.

Answer: Individuals vary around the mean and may fall below a safety threshold.