Sampling and confidence intervals - Paper 3

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

A community transport project combines survey design, estimates and careful reporting. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A town has 2,400 residents in its east zone and 1,600 in its west zone. Plan a proportional random sample of 100. [6 marks]

(a) Give the stratum sizes and selection method. [3 marks]

(b) Why can surveying 100 people at the central bus stop distort a town-wide estimate? [3 marks]

2. The selected sample contains 60 east and 40 west residents. Only 30 east and all 40 west residents reply. [6 marks]

(a) Find the east-zone proportion among respondents and compare it with the town proportion. [3 marks]

(b) Explain what this reveals without assuming a direction of opinion bias. [3 marks]

3. A separate representative random sample of 625 residents has 375 regular public-transport users. [6 marks]

(a) Find the estimate and estimated standard error. [3 marks]

(b) Construct a 95% interval for the town’s regular-use proportion. [3 marks]

4. Normal waiting times in a controlled model have mean 12 minutes and SD 6 minutes. A mean is formed from 36 independent waits. [6 marks]

(a) Find the model for the mean. [3 marks]

(b) Find the probability that the mean is below 10.5 minutes. [3 marks]

5. A large random independent waiting-time sample of 100 has mean 11.8 minutes and sample standard deviation 5 minutes. Population spread is unknown. Use a large-sample normal approximation. [6 marks]

(a) Construct an approximate 90% confidence interval for the population mean. [3 marks]

(b) The underlying wait distribution is right-skewed. Explain the normal-model justification and its limitation. [3 marks]

6. Assume a population proportion $p = 0.6$ and $n = 625$ independent observations. [6 marks]

(a) Find the approximate probability that the sample proportion lies between 0.56 and 0.64. [3 marks]

(b) Explain why this probability calculation uses 0.6 inside the standard deviation. [3 marks]

7. An online press report describes a 95% mean interval $(10.9, 12.7)$ minutes. [6 marks]

(a) Correct “95% of residents wait between 10.9 and 12.7 minutes.” [3 marks]

(b) Can the interval establish an average wait below 12 minutes? [3 marks]

8. Planners require a 95% proportion margin at most 0.025 with no defensible prior estimate. [6 marks]

(a) Find the conservative sample size. [3 marks]

(b) If the expected response rate is 80%, estimate the contact budget and state one limitation. [3 marks]

9. Two separate 95% support intervals are east $(0.55, 0.63)$ and west $(0.57, 0.65)$. [6 marks]

(a) Recover each estimate and margin. [3 marks]

(b) Evaluate “Their overlap proves there is no difference.” [3 marks]

10. A report claims “The proportion will still be 60% in five years because today’s interval is narrow.” [6 marks]

(a) Identify the unsupported step. [3 marks]

(b) Write a more responsible conclusion. [3 marks]