Sampling and confidence intervals - Paper 2

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Interpret study records, recover missing quantities and compare estimation procedures. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A report on bus use supplies the following record. [6 marks]

One survey record

Selected	Responded	Respondents using bus
300	240	144

(a) Find the proportion using the bus among respondents. [3 marks]

(b) Explain why $\frac{144}{300}$ does not estimate the same respondent proportion. [3 marks]

2. A 95% known-spread mean confidence interval is $(31.08, 34.22)$. [6 marks]

(a) Find the sample mean and standard error. [3 marks]

(b) A proposed population mean is 33. Can this interval prove that value? [3 marks]

3. A sample mean has standard error 0.75 kg, based on 64 independent observations. [6 marks]

(a) Recover the population standard deviation. [3 marks]

(b) What sample size gives standard error at most 0.5 kg? [3 marks]

4. Normal lifetimes have mean 80 hours and SD 16 hours. Samples contain 64 independent items. [6 marks]

(a) Find the probability that a sample mean exceeds 83 hours. [3 marks]

(b) Would this probability also describe one randomly chosen lifetime exceeding 83 hours? Explain. [3 marks]

5. Study A has known population variance 100 and sample size 25. Study B has variance 144 and sample size 36. [6 marks]

(a) Compare the standard errors of their means. [3 marks]

(b) At identical confidence levels, compare interval widths and centres. [3 marks]

6. For an independent binary sample, X counts successes among 200 trials with $p = 0.25$. [6 marks]

(a) Find the means and variances of X and the sample proportion. [3 marks]

(b) Explain why the sample proportion has a discrete distribution. [3 marks]

7. An independently obtained proportion estimate is 0.64 and its estimated standard error is 0.02. A suitable normal approximation is assumed. [6 marks]

(a) Give the 98% interval using critical value 2.326. [3 marks]

(b) Convert its margin into percentage points. [3 marks]

8. A study using a correct 95% procedure produces 1,000 independent intervals. [6 marks]

(a) Find the expected captures and expected misses. [3 marks]

(b) Must exactly 950 intervals capture the parameter? [3 marks]

9. A clinic numbers patient records in repeated weekday order and chooses every seventh record after a random start. [6 marks]

(a) Identify a possible problem. [3 marks]

(b) Describe an alternative preserving a probability basis. [3 marks]

10. A project can afford at most 500 completed responses. There is no prior proportion estimate. [6 marks]

(a) Find its conservative 95% margin. [3 marks]

(b) Can it promise a margin at most 0.04 under this planning formula? If not, give the required size. [3 marks]