Sampling and confidence intervals - Paper 2

10 questions | 60 marks | 75 minutes

Interpret study records, recover missing quantities and compare estimation procedures. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Equivalent correct methods earn the corresponding marks.

1. A report on bus use supplies the following record. [6 marks]

One survey record

Selected	Responded	Respondents using bus
300	240	144

(a) Find the proportion using the bus among respondents. [3 marks]

$\frac{144}{240} = 0.6$.

Answer: 0.6.

(b) Explain why $\frac{144}{300}$ does not estimate the same respondent proportion. [3 marks]

The 60 missing responses are not known non-users.

Answer: It counts selected people who both responded and use the bus.

2. A 95% known-spread mean confidence interval is $(31.08, 34.22)$. [6 marks]

(a) Find the sample mean and standard error. [3 marks]

Midpoint $= 32.65$; margin $= 1.57$.

Divide the margin by 1.96.

Answer: Mean 32.65; SE 0.801.

(b) A proposed population mean is 33. Can this interval prove that value? [3 marks]

An interval supports a range of parameter values.

Answer: No. It is compatible with the interval but not proved exactly true.

3. A sample mean has standard error 0.75 kg, based on 64 independent observations. [6 marks]

(a) Recover the population standard deviation. [3 marks]

$0.75 \times \sqrt{64} = 6$.

Answer: 6 kg.

(b) What sample size gives standard error at most 0.5 kg? [3 marks]

Require $n \geq \left(\frac{6}{0.5}\right)^2 = 144$.

Answer: 144.

4. Normal lifetimes have mean 80 hours and SD 16 hours. Samples contain 64 independent items. [6 marks]

(a) Find the probability that a sample mean exceeds 83 hours. [3 marks]

The sample-mean SD is $\frac{16}{8} = 2$.

The standardised boundary is 1.5.

Answer: 0.0668

(b) Would this probability also describe one randomly chosen lifetime exceeding 83 hours? Explain. [3 marks]

Individual and sample-mean distributions have different spreads.

Answer: No; one lifetime has SD 16, not 2.

5. Study A has known population variance 100 and sample size 25. Study B has variance 144 and sample size 36. [6 marks]

(a) Compare the standard errors of their means. [3 marks]

A: $\frac{\sqrt{100}}{\sqrt{25}} = 2$. B: $\frac{\sqrt{144}}{\sqrt{36}} = 2$.

Answer: Both are 2.

(b) At identical confidence levels, compare interval widths and centres. [3 marks]

Equal standard errors and critical values give equal widths, but observed means can differ.

Answer: Widths are equal; centres need not be equal.

6. For an independent binary sample, X counts successes among 200 trials with $p = 0.25$. [6 marks]

(a) Find the means and variances of X and the sample proportion. [3 marks]

Use np and $np(1-p)$, then divide X by 200.

Answer: $E(X) = 50;\ \mathrm{Var}(X) = 37.5;\ E(\hat{p}) = 0.25;\ \mathrm{Var}(\hat{p}) = 0.0009375.$

(b) Explain why the sample proportion has a discrete distribution. [3 marks]

The success count is an integer.

Answer: It can only be $0, \frac{1}{200}, \frac{2}{200}, \ldots, 1.$

7. An independently obtained proportion estimate is 0.64 and its estimated standard error is 0.02. A suitable normal approximation is assumed. [6 marks]

(a) Give the 98% interval using critical value 2.326. [3 marks]

Margin $= 2.326 \times 0.02 = 0.04652.$

Answer: $(0.5935, 0.6865)$

(b) Convert its margin into percentage points. [3 marks]

Multiply the proportion margin by 100.

Answer: 4.652 percentage points.

8. A study using a correct 95% procedure produces 1,000 independent intervals. [6 marks]

(a) Find the expected captures and expected misses. [3 marks]

Multiply the number of intervals by 0.95 and 0.05.

Answer: 950 captures; 50 misses.

(b) Must exactly 950 intervals capture the parameter? [3 marks]

Expected value is not a guaranteed realised count.

Answer: No. The number varies randomly.

9. A clinic numbers patient records in repeated weekday order and chooses every seventh record after a random start. [6 marks]

(a) Identify a possible problem. [3 marks]

A random start does not necessarily remove a periodicity problem.

Answer: The interval can repeatedly select one weekday, which may differ systematically from the others.

(b) Describe an alternative preserving a probability basis. [3 marks]

The method should cover the target population appropriately.

Answer: Randomly select distinct labels from the full list, or stratify by weekday and sample randomly within each group.

10. A project can afford at most 500 completed responses. There is no prior proportion estimate. [6 marks]

(a) Find its conservative 95% margin. [3 marks]

Use $p_0 = 0.5$ and $n = 500$.

Answer: 0.0438

(b) Can it promise a margin at most 0.04 under this planning formula? If not, give the required size. [3 marks]

The calculated margin is about 0.0438.

$\frac{1.96^2 \times 0.25}{0.04^2} = 600.25$; round up.

Answer: No; at least 601 completed responses are required.