Sampling and confidence intervals - Paper 1

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Build from sampling design to normal models, interval estimation and interpretation. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A school has 540 junior and 360 senior students. It wants a random sample of 60 to estimate a school-wide proportion. [6 marks]

(a) Give proportional stratum sizes and describe selection. [3 marks]

(b) The school instead asks the first 60 students at a sports meeting. Explain one limitation. [3 marks]

2. Independent normal packet masses have population mean 250 g and known standard deviation 9 g. A sample contains 36 packets. [6 marks]

(a) State the distribution of the sample mean. [3 marks]

(b) Find $P(247 < \bar{X} < 253)$. [3 marks]

3. A random independent sample of 81 cables has mean length 102.3 cm. Treat population standard deviation as known at 4.5 cm; lengths are normal. [6 marks]

(a) Find a 95% confidence interval for the population mean. [3 marks]

(b) Explain what the confidence level means. [3 marks]

4. In a random sample of 500 residents, 290 support a new service. [6 marks]

(a) Calculate the estimate and estimated standard error. [3 marks]

(b) Find a 95% normal-approximation confidence interval. [3 marks]

5. Two reports use the same mean, sample size and known population standard deviation. One reports 90% confidence and one 99%. [6 marks]

(a) Calculate the ratio of 99% width to 90% width. [3 marks]

(b) Explain the precision trade-off. [3 marks]

6. A population standard deviation is 18 units. A study requires a 95% mean margin no greater than 2 units. [6 marks]

(a) Find the minimum number of independent valid observations. [3 marks]

(b) If each observation costs AUD 4, give the minimum data-collection cost. [3 marks]

7. Assume a binary population has $p = 0.3$ and independent sampling gives $n = 400$. [6 marks]

(a) Find the mean and standard deviation of the sample proportion. [3 marks]

(b) Check the expected-count screening conditions for normal approximation. [3 marks]

8. A poll reports a 95% interval of $(0.48, 0.56)$ for a support proportion. [6 marks]

(a) Recover its centre and margin. [3 marks]

(b) Evaluate the statement “A majority is established.” [3 marks]

9. A researcher wants a 95% proportion interval with margin at most 0.03 and has no prior estimate. [6 marks]

(a) Find a conservative minimum sample size. [3 marks]

(b) Explain why the planning value 0.5 is conservative. [3 marks]

10. A website poll receives 12,000 voluntary responses and reports a tiny margin for all state residents. [6 marks]

(a) Identify two issues the computed sampling margin may not capture. [3 marks]

(b) Suggest one defensible improvement and a limitation to report. [3 marks]