Sampling and confidence intervals - Paper 1

10 questions | 60 marks | 75 minutes

Build from sampling design to normal models, interval estimation and interpretation. Use supplied two-sided critical values: 90% 1.645, 95% 1.96, 98% 2.326 and 99% 2.576. Retain intermediate precision; give final probabilities to four decimal places and state assumptions.

Equivalent correct methods earn the corresponding marks.

1. A school has 540 junior and 360 senior students. It wants a random sample of 60 to estimate a school-wide proportion. [6 marks]

(a) Give proportional stratum sizes and describe selection. [3 marks]

There are 900 students. Multiply each stratum fraction by 60.

Use distinct random labels from complete lists.

Answer: 36 juniors and 24 seniors, chosen randomly within each stratum.

(b) The school instead asks the first 60 students at a sports meeting. Explain one limitation. [3 marks]

The frame does not give every school student the intended chance of selection.

Answer: The convenience sample can over-represent students attending sport.

2. Independent normal packet masses have population mean 250 g and known standard deviation 9 g. A sample contains 36 packets. [6 marks]

(a) State the distribution of the sample mean. [3 marks]

Centre $=250$; standard error $=\frac{9}{6}=1.5$ g.

Answer: $\bar{X} \sim N(250, 1.5^2)$.

(b) Find $P(247 < \bar{X} < 253)$. [3 marks]

The bounds standardise to -2 and 2.

Evaluate the central normal probability.

Answer: 0.9545

3. A random independent sample of 81 cables has mean length 102.3 cm. Treat population standard deviation as known at 4.5 cm; lengths are normal. [6 marks]

(a) Find a 95% confidence interval for the population mean. [3 marks]

Standard error $= \frac{4.5}{9} = 0.5$ cm.

Margin $= 1.96 \times 0.5 = 0.98$ cm.

Answer: $(101.32, 103.28)$

(b) Explain what the confidence level means. [3 marks]

Confidence concerns the interval method, not the proportion of individual cables inside these bounds.

Answer: About 95% of intervals from repeated samples using this procedure contain the fixed population mean.

4. In a random sample of 500 residents, 290 support a new service. [6 marks]

(a) Calculate the estimate and estimated standard error. [3 marks]

$\frac{290}{500} = 0.58.$

Use the square root of $\frac{0.58 \times 0.42}{500}$.

Answer: Estimate 0.58; estimated SE 0.0221.

(b) Find a 95% normal-approximation confidence interval. [3 marks]

Multiply estimated SE by 1.96.

Add and subtract from 0.58; observed counts 290 and 210 support the approximation.

Answer: $(0.5367, 0.6233)$

5. Two reports use the same mean, sample size and known population standard deviation. One reports 90% confidence and one 99%. [6 marks]

(a) Calculate the ratio of 99% width to 90% width. [3 marks]

All factors except the critical value cancel.

Take $\frac{2.576}{1.645}$.

Answer: 1.566

(b) Explain the precision trade-off. [3 marks]

The sample information has not increased.

Answer: The 99% interval is wider: greater long-run coverage with less precision.

6. A population standard deviation is 18 units. A study requires a 95% mean margin no greater than 2 units. [6 marks]

(a) Find the minimum number of independent valid observations. [3 marks]

$n \geq \left(\frac{1.96 \times 18}{2}\right)^2 = 311.1696$; round up.

Answer: 312

(b) If each observation costs AUD 4, give the minimum data-collection cost. [3 marks]

$312 \times 4 = 1{,}248$.

This excludes other costs and assumes every observation is valid.

Answer: AUD 1,248.

7. Assume a binary population has $p = 0.3$ and independent sampling gives $n = 400$. [6 marks]

(a) Find the mean and standard deviation of the sample proportion. [3 marks]

Use p and the square root of $\frac{p(1-p)}{n}$.

Answer: Mean 0.3; SD 0.0229.

(b) Check the expected-count screening conditions for normal approximation. [3 marks]

The design and independence assumptions must also hold.

Answer: Expected successes 120 and failures 280; both are sufficiently large for the stated screening rule.

8. A poll reports a 95% interval of $(0.48, 0.56)$ for a support proportion. [6 marks]

(a) Recover its centre and margin. [3 marks]

Take the midpoint and half-width.

Answer: Centre 0.52; margin 0.04 (four percentage points).

(b) Evaluate the statement “A majority is established.” [3 marks]

Compatibility with both sides prevents the stated conclusion.

Answer: Not established by this interval: it includes support below and above 0.5.

9. A researcher wants a 95% proportion interval with margin at most 0.03 and has no prior estimate. [6 marks]

(a) Find a conservative minimum sample size. [3 marks]

Use $p_0 = 0.5$: $n \geq \frac{1.96^2(0.25)}{0.03^2} = 1067.111\ldots$; round up.

Answer: 1068

(b) Explain why the planning value 0.5 is conservative. [3 marks]

It gives the largest variance factor and thus the largest planned sample.

Answer: $p(1-p) = 0.25 - (p-0.5)^2 \leq 0.25.$

10. A website poll receives 12,000 voluntary responses and reports a tiny margin for all state residents. [6 marks]

(a) Identify two issues the computed sampling margin may not capture. [3 marks]

Website users may not represent all residents; choosing to reply can depend on the opinion.

Answer: Coverage bias and self-selection/non-response bias.

(b) Suggest one defensible improvement and a limitation to report. [3 marks]

Increasing only the number of website replies is not enough.

Answer: Draw a probability sample from an appropriate resident frame, follow up non-response, and report remaining response or coverage limitations.