Sample means and the central-limit idea

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. For $\mu = 72, \sigma = 18, n = 36$, find the mean and variance of $\bar{X}$. [2 marks]

$E(\bar{X}) = 72;\ \mathrm{Var}(\bar{X}) = \frac{18^2}{36} = 9.$

Answer: 72 and 9.

2. Normal observations have mean 100 and standard deviation 20. Model a mean of 16 independent observations. [2 marks]

The standard error is $\frac{20}{\sqrt{16}} = 5.$

Answer: $\bar{X} \sim N(100, 5^2).$

3. For the previous model, find $P(\bar{X} > 110)$. [2 marks]

$z = \frac{110 - 100}{5} = 2.$

$1 - \Phi(2) = 0.022750\ldots.$

Answer: Approximately 0.0228.

4. Why need a mean of four highly right-skewed observations not be approximately normal? [2 marks]

The central limit theorem is a limiting result, not a guarantee for every small sample.

Answer: A sample of four may be too small to average away strong skewness.

5. What changes as independent sample size grows for a non-normal finite-variance population? [2 marks]

Its standard deviation is $\frac{\sigma}{\sqrt{n}}$; its expectation is μ.

Answer: The mean distribution tends towards normality and becomes narrower; its centre remains the population mean.

6. Why can 100 consecutive readings from one drifting sensor fail independence? [2 marks]

A large reading count is not evidence that the observations are independent.

Answer: Adjacent readings can share drift and be correlated.