Practice test 6 - connected applications

Name: _____ Date: _____

10 questions | 47 marks | 75 minutes

A more demanding paper joining exact density work, normal parameter inference and independent-sample reasoning.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A model has density $f(x) = k(1 - x^2)$ for $-1 \le x \le 1$, zero elsewhere. [5 marks]

(a) Find k and justify nonnegativity. [2 marks]

(b) Find $P(|X| < 0.5)$. [3 marks]

2. For the density in Question 1, [5 marks]

(a) find $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$. [3 marks]

(b) find the mean and variance of $Y = 4X + 7$. [2 marks]

3. For the same model, [5 marks]

(a) show that its interior CDF is $\frac{1}{2}+\frac{3}{4}x-\frac{1}{4}x^3$. [3 marks]

(b) Use symmetry to find the median, then explain why the 90th percentile must be positive. [2 marks]

4. A normal model has its 25th percentile at 42 and its 75th percentile at 58. [5 marks]

(a) Find its mean and SD. [3 marks]

(b) Find its probability of lying below 30. [2 marks]

5. For a normal variable with mean 100, $P(X > 112) = 0.10$. [5 marks]

(a) Find its SD. [2 marks]

(b) Find a symmetric interval about 100 containing 98% of values. [3 marks]

6. Independent $X \sim N(50, 16)$ and $Y \sim N(52, 9)$ model two independent measurements. [5 marks]

(a) Find the distribution of $X - Y$. [2 marks]

(b) Find the probability that the two measurements differ by less than 3. [3 marks]

7. An independent random sample is taken from a normal population with SD 12. [4 marks]

(a) Find the smallest sample size making the standard error at most 2. [2 marks]

(b) For that sample size and population mean 80, find $P(\text{sample mean} > 83)$. [2 marks]

8. An independent random sample of 64 is taken from a non-normal population with mean 40 and finite SD 8. [4 marks]

(a) Approximate $P(38 < \text{sample mean} < 41)$. [2 marks]

(b) State two limitations of interpreting this probability. [2 marks]

9. Let X_1 and X_2 be independent observations from a normal population with mean 10 and variance 4. Compare $A = X_1 + X_2$ and $B = 2X_1$. [5 marks]

(a) Find both distributions. [2 marks]

(b) Which has the larger probability of exceeding 24? Give both probabilities. [3 marks]

10. A report rounds a calculated normal tail probability of 0.00047 to two decimal places and reports zero. [4 marks]

(a) Explain why saying the event is impossible is not justified. [2 marks]

(b) Give a clearer probability report and one modelling qualification. [2 marks]