Practice test 5 - modelling and interpretation

Name: _____ Date: _____

10 questions | 43 marks | 75 minutes

Work with a two-piece density, compare alternative normal decisions, and evaluate modelling assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A density equals kx on $[0, 1]$ and $k(2 - x)$ on $(1, 2]$, and is zero outside $[0, 2]$. [4 marks]

(a) Find k. [2 marks]

(b) Find $P(0.5 < X < 1.5)$. [2 marks]

2. Use the two-piece density from Question 1. [5 marks]

(a) Give its CDF for x between 0 and 1, then for x between 1 and 2. [3 marks]

(b) Find $P(X > 1.5 \mid X > 1)$. [2 marks]

3. The same distribution is symmetric around $x = 1$. [5 marks]

(a) Find $\mathrm{E}(X)$ by symmetry, then show $\mathrm{E}(X^2) = \frac{7}{6}$. [3 marks]

(b) Find its variance and standard deviation. [2 marks]

4. A normally distributed dimension has mean 30 mm and SD 0.2 mm. Parts are accepted when between 29.7 and 30.4 mm. [5 marks]

(a) Find the probability of acceptance. [2 marks]

(b) Which rejection tail is larger? Quantify both. [3 marks]

5. A normally distributed demand has mean 200 and SD 30. A business wants stock sufficient on 98% of days. [4 marks]

(a) Find the required model cut-off. [2 marks]

(b) If stock must be a whole number, what is the smallest sufficient stock under this model? [2 marks]

6. Independent normal variables satisfy $A \sim N(12, 4)$, $B \sim N(8, 1)$ and $C \sim N(10, 9)$. [4 marks]

(a) Find the distribution of $A + B - C$. [2 marks]

(b) Find $P(A + B > C)$. [2 marks]

7. Normal packet masses have mean 100 g and SD 5 g. A random independent sample contains 100 packets. [4 marks]

(a) Find the probability that the sample mean is below 99 g. [2 marks]

(b) Would the probability that one packet is below 99 g be as small? Calculate it. [2 marks]

8. A population is strongly right-skewed. Its mean is 6 and SD is 4. [4 marks]

(a) For a random independent sample of 400, give the approximate mean distribution. [2 marks]

(b) Does this make the original population normal? [2 marks]

9. A factory combines equal numbers of outputs from machines with means 10 and 30. Each machine output is normal with SD 1. [4 marks]

(a) Explain why the combined individual-output distribution need not be normal. [2 marks]

(b) What is the combined population mean? [2 marks]

10. A quality-control report says: “The model probability of exceeding the limit is 0.01, so exactly one of the next 100 outputs will exceed it.” [4 marks]

(a) Explain the error. [2 marks]

(b) State the expected number exceeding the limit in 250 outputs. [2 marks]