Practice test 4 - reverse calculations and comparisons

Name: _____ Date: _____

10 questions | 42 marks | 65 minutes

Develop density and CDF reasoning, infer normal parameters, and distinguish correlated readings from independent samples.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A continuous model has density $f(x) = kx^2$ on $[0, 3]$. [4 marks]

(a) Find k. [2 marks]

(b) Find $P(X > 2)$. [2 marks]

2. For the model $f(x) = \frac{x^2}{9}$ on $[0, 3]$, [4 marks]

(a) find the 80th percentile. [2 marks]

(b) find $P(X > 2 \mid X > 1)$. [2 marks]

3. For density $\frac{x^2}{9}$ on $[0, 3]$, [5 marks]

(a) find the first and second moments. [3 marks]

(b) find the variance and the variance of $Y = 2X - 1$. [2 marks]

4. A machine output is modelled as normal. Its 5th and 95th percentiles are 28 and 44. [5 marks]

(a) Find its mean. [2 marks]

(b) Find its standard deviation. [3 marks]

5. $X \sim N(60, 25)$. [4 marks]

(a) Find $P(X < 55 \text{ or } X > 70)$. [2 marks]

(b) Find the cut-off c such that $P(X > c) = 0.025$. [2 marks]

6. A normal test score has mean 100 and SD 15. Two students score 118 and 85. [4 marks]

(a) Find both z-scores. [2 marks]

(b) Find the proportion of scores between the two students. [2 marks]

7. Independent normal X and Y have means 8 and 3, and variances 1 and 4. [4 marks]

(a) Find the mean and variance of $W = 3X - 2Y$. [2 marks]

(b) Find $P(W < 23)$. [2 marks]

8. A normal population has mean 250 and SD 20. Independent samples of 16 observations are taken. [4 marks]

(a) Find the probability that a sample mean lies between 245 and 260. [2 marks]

(b) Find the probability that a total of 16 observations exceeds 4160. [2 marks]

9. A lab records the same object ten times with an instrument having a shared unknown calibration error. [4 marks]

(a) Why might the ten readings fail an independence assumption? [2 marks]

(b) Would dividing a claimed population SD by the square root of ten automatically account for that calibration error? [2 marks]

10. A density curve has 40% of its area below 2 and 75% below 5. [4 marks]

Cumulative probability information

x	$F(x)$
2	0.40
5	0.75

(a) Find $P(2 < X < 5)$. [2 marks]

(b) Find $P(X < 5 \mid X > 2)$. [2 marks]