Practice test 3 - applications and assumptions

Name: _____ Date: _____

10 questions | 42 marks | 65 minutes

Connect polynomial densities with practical normal models and compare totals, averages and model choices.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A proposed density is $f(x) = kx(1 - x)$ on $[0, 1]$. [4 marks]

(a) Find k and justify nonnegativity. [2 marks]

(b) Find $P(0.2 < X < 0.8)$. [2 marks]

2. For density $6x(1 - x)$ on $[0, 1]$, [5 marks]

(a) find $\mathrm{E}(X)$. [2 marks]

(b) find its variance. [3 marks]

3. Its interior CDF is $F(x) = 3x^2 - 2x^3$. [4 marks]

(a) Check the two endpoint CDF values. [2 marks]

(b) Explain why 0.5 is a median and find $P(X > 0.5)$. [2 marks]

4. A normal model describes delivery time with mean 36 minutes and SD 4 minutes. [4 marks]

(a) Find the chance a delivery takes between 30 and 40 minutes. [2 marks]

(b) Find a time exceeded by only 5% of deliveries. [2 marks]

5. A normal score distribution has 16th percentile 62 and 84th percentile 78. [5 marks]

(a) Find the mean. [2 marks]

(b) Calculate the SD using the exact standard-normal percentiles. [3 marks]

6. Independent masses $X \sim N(30, 4)$ and $Y \sim N(20, 9)$ are measured in grams. [4 marks]

(a) Find the distribution of $2X + Y$. [2 marks]

(b) Find $P(2X + Y > 90)$. [2 marks]

7. Nine independently measured cable lengths each follow a normal model with mean 2 m and SD 0.1 m. [4 marks]

(a) Find the mean and SD of their total length. [2 marks]

(b) Find the chance the total is at least 18.3 m. [2 marks]

8. The same nine cable lengths are averaged. [4 marks]

(a) Find the standard error of their mean. [2 marks]

(b) Explain why adding 0.3 m to the total corresponds to adding only $\frac{1}{30}$ m to the average. [2 marks]

9. A farm models a nonnegative quantity as normal with mean 1 and SD 2. [4 marks]

(a) Find the normal-model probability of a negative value. [2 marks]

(b) What does that result suggest about the model? [2 marks]

10. A researcher compares a random sample of 25 observations with a random sample of 100 from the same population. [4 marks]

(a) Compare the standard errors. [2 marks]

(b) Explain why this alone does not prove either sample mean is close to the true mean. [2 marks]