Marking guide | Continuous probability and normal distributions

Practice test 3 - applications and assumptions

10 questions | 42 marks | 65 minutes

Connect polynomial densities with practical normal models and compare totals, averages and model choices.

Equivalent correct methods earn the corresponding marks.

1. A proposed density is $f(x) = kx(1-x)$ on $[0, 1]$. [4 marks]

(a) Find k and justify nonnegativity. [2 marks]

$\int_0^1 x(1-x)\,dx = \frac{1}{6}$; both factors are nonnegative on $[0, 1]$.

Answer: $k = 6$.

(b) Find $P(0.2 < X < 0.8)$. [2 marks]

Use antiderivative $3x^2 - 2x^3$; values are 0.896 and 0.104.

Answer: 0.792.

2. For density $6x(1-x)$ on $[0, 1]$, [5 marks]

(a) find $\mathrm{E}(X)$. [2 marks]

By symmetry around $\frac{1}{2}$, or integrate $6x^2(1-x)$ to obtain $\frac{1}{2}$.

Answer: $\frac{1}{2}$.

(b) find its variance. [3 marks]

$\mathrm{E}(X^2) = 6(\frac{1}{4} - \frac{1}{5}) = \frac{3}{10}$.

$\frac{3}{10} - (\frac{1}{2})^2 = \frac{1}{20}$.

Answer: $\frac{1}{20}$.

3. Its interior CDF is $F(x) = 3x^2 - 2x^3$. [4 marks]

(a) Check the two endpoint CDF values. [2 marks]

Direct substitution gives the cumulative limits.

Answer: $F(0) = 0$ and $F(1) = 1$.

(b) Explain why 0.5 is a median and find $P(X > 0.5)$. [2 marks]

$3(\frac{1}{4}) - 2(\frac{1}{8}) = \frac{1}{2}$; take the complement.

Answer: $F(0.5) = 0.5$; upper probability 0.5.

4. A normal model describes delivery time with mean 36 minutes and SD 4 minutes. [4 marks]

(a) Find the chance a delivery takes between 30 and 40 minutes. [2 marks]

The standardised bounds are -1.5 and 1.

Answer: 0.7745.

(b) Find a time exceeded by only 5% of deliveries. [2 marks]

Use lower-tail area 0.95, not 0.05.

Answer: 42.5794 minutes.

5. A normal score distribution has 16th percentile 62 and 84th percentile 78. [5 marks]

(a) Find the mean. [2 marks]

The two percentiles are symmetric, so average the endpoints.

Answer: 70.

(b) Calculate the SD using the exact standard-normal percentiles. [3 marks]

$78 = 70 + \Phi^{-1}(0.84)\sigma$.

Answer: 8.0446.

6. Independent masses $X \sim N(30, 4)$ and $Y \sim N(20, 9)$ are measured in grams. [4 marks]

(a) Find the distribution of $2X + Y$. [2 marks]

Mean $2(30) + 20 = 80$; variance $4(4) + 9 = 25$.

Answer: $N(80, 25)$.

(b) Find $P(2X + Y > 90)$. [2 marks]

Use the mean 80 and SD 5 to obtain $z = 2$.

Answer: 0.0228

7. Nine independently measured cable lengths each follow a normal model with mean 2 m and SD 0.1 m. [4 marks]

(a) Find the mean and SD of their total length. [2 marks]

Mean is $9(2)$; variance is $9(0.01) = 0.09$.

Answer: 18 m and 0.3 m.

(b) Find the chance the total is at least 18.3 m. [2 marks]

The standardised cut-off is 1.

Answer: 0.1587

8. The same nine cable lengths are averaged. [4 marks]

(a) Find the standard error of their mean. [2 marks]

$\frac{0.1}{\sqrt{9}} = \frac{0.1}{3}$.

Answer: $\frac{1}{30} \approx 0.03333$ m.

(b) Explain why adding 0.3 m to the total corresponds to adding only $\frac{1}{30}$ m to the average. [2 marks]

The event $\text{total} > 18.3$ is the same as $\text{mean} > \frac{18.3}{9}$, whose bound is $2 + \frac{1}{30}$.

Answer: The average divides the total by nine.

9. A farm models a nonnegative quantity as normal with mean 1 and SD 2. [4 marks]

(a) Find the normal-model probability of a negative value. [2 marks]

Standardise zero: $\frac{0-1}{2} = -0.5$.

Answer: 0.3085

(b) What does that result suggest about the model? [2 marks]

It assigns substantial probability to physically impossible negative values; inspect another distribution or the data.

Answer: The normal model may be poor for this quantity.

10. A researcher compares a random sample of 25 observations with a random sample of 100 from the same population. [4 marks]

(a) Compare the standard errors. [2 marks]

$\frac{\sqrt{25}}{\sqrt{100}} = \frac{1}{2}$.

Answer: The $n = 100$ SE is half the $n = 25$ SE.

(b) Explain why this alone does not prove either sample mean is close to the true mean. [2 marks]

An unusual sample remains possible; representativeness and sampling assumptions also matter.

Answer: SE describes repeated-sampling variability, not certainty about one sample.