Practice test 2 - cumulative models and cut-offs

Name: _____ Date: _____

10 questions | 42 marks | 65 minutes

Use changing density shapes, CDFs, reverse normal calculations and weighted measurements.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Let $f(x) = k(2 - x)$ on $[0, 2]$, zero elsewhere. [4 marks]

(a) Find k. [2 marks]

(b) Find $P(X < 1)$. [2 marks]

2. For density $1 - \frac{x}{2}$ on $[0, 2]$, [5 marks]

(a) find the CDF on the whole real line. [3 marks]

(b) find the median. [2 marks]

3. A density equals c on $[0, 1]$ and $2c$ on $(1, 2]$, and is zero elsewhere. [4 marks]

(a) Find c and $P(X < 0.5)$. [2 marks]

(b) Does the CDF jump at $x = 1$? Explain. [2 marks]

4. U is uniform on $[-2, 4]$. [5 marks]

(a) Find $\mathrm{E}(U)$, $\mathrm{Var}(U)$ and $\mathrm{SD}(U)$. [3 marks]

(b) Find the mean and variance of $V = 3U + 2$. [2 marks]

5. $X \sim N(24, 2.25)$ models filling time in seconds. [4 marks]

(a) Find $P(X > 27)$. [2 marks]

(b) Find the central 80% interval. [2 marks]

6. A normal model has known mean 100. Its 90th percentile is 112. [4 marks]

(a) Find its standard deviation. [2 marks]

(b) State its 10th percentile. [2 marks]

7. Independent $X \sim N(10, 4)$ and $Y \sim N(5, 9)$. [4 marks]

(a) Find the distribution of $2X - Y$. [2 marks]

(b) Find $P(2X - Y < 10)$. [2 marks]

8. A normal population has mean 60 and SD 10. An independent random sample of 25 values is selected. [4 marks]

(a) Find the SE and $P(\text{sample mean} > 64)$. [2 marks]

(b) Would $P(\text{an individual value} > 64)$ be the same? Calculate it. [2 marks]

9. A non-normal population has mean 18 and finite SD 6. [4 marks]

(a) For $n = 100$ independent random observations, give an approximate distribution of the sample mean. [2 marks]

(b) Identify one feature that could make convergence to normality slower. [2 marks]

10. A sensor rounds continuous measurements to the nearest whole number. The underlying X has density $2x$ on $[0, 1]$. [4 marks]

(a) Find $P(\text{the displayed value is } 1)$. [2 marks]

(b) Explain why the displayed value can have positive probability although $P(X = 1) = 0$. [2 marks]