Practice test 1 - areas and normal models

Name: _____ Date: _____

10 questions | 41 marks | 65 minutes

Build from valid densities and interval areas to normal cut-offs, independent totals and sample means.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A model has density $f(x) = kx$ for $0 \le x \le 2$, and zero elsewhere. [4 marks]

(a) Find k and check that this gives a valid density. [2 marks]

(b) Find $P(0.5 < X < 1.5)$. [2 marks]

2. A bus waiting time W is uniform on $[1, 7]$ minutes. [4 marks]

(a) State the density and find $P(W < 3)$. [2 marks]

(b) Given that $W > 3$, find the chance $W > 5$. [2 marks]

3. A continuous CDF equals x^3 on $[0, 1]$, with outer values zero and one. [4 marks]

(a) Find the density and $P(0.2 < X < 0.8)$. [2 marks]

(b) Find the median. [2 marks]

4. X has density $2x$ on $[0, 1]$. [5 marks]

(a) Find $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$. [3 marks]

(b) Find $\mathrm{Var}(X)$ and the standard deviation. [2 marks]

5. $Y \sim N(50, 16)$, using mean-variance notation. [4 marks]

(a) State the standard deviation and standardise $Y = 56$. [2 marks]

(b) Find $P(46 < Y < 56)$ to four decimals. [2 marks]

6. A normal component length has mean 80 mm and SD 3 mm. [4 marks]

(a) Find the length exceeded by 10% of components. [2 marks]

(b) Explain why the 10th percentile would answer a different question. [2 marks]

7. Independent $A \sim N(20, 9)$ and $B \sim N(12, 4)$ model two processing times in minutes. [4 marks]

(a) Find the distribution of $T = A + B$. [2 marks]

(b) Find $P(T > 36)$. [2 marks]

8. Independent packet masses come from a normal population with mean 500 g and SD 12 g. A sample contains 9 packets. [4 marks]

(a) Find the distribution of the sample mean. [2 marks]

(b) Find $P(\text{sample mean} < 496)$. [2 marks]

9. A student models two times as normal and then adds their variances. [4 marks]

(a) What further assumption is needed? [2 marks]

(b) If A and B are the same random reading X with $\mathrm{Var}(X) = 4$, compare $\mathrm{Var}(A + B)$ with the claimed 8. [2 marks]

10. A company estimates the mean life of a product using a large sample of volunteers who report a failed product. [4 marks]

(a) Explain why the large sample may still be biased. [2 marks]

(b) State one change that would support a valid sample-mean model. [2 marks]