Independent sums and sample means

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Independent $X \sim N(10, 4)$, $Y \sim N(5, 9)$. Find the distribution of $X + Y$. [2 marks]

Means add and independent variances add.

Answer: $N(15, 13)$.

2. A normal population has mean 80 and SD 12. Find the distribution of a mean from 9 independent observations. [2 marks]

The variance is $\frac{144}{9} = 16$.

Answer: $N(80, 16)$.

3. For that sample mean, find $P(\text{mean} > 84)$. [2 marks]

Standard error is 4; $z = \frac{84 - 80}{4} = 1$.

Answer: 0.1587.

4. If X has variance 4, compare $\text{Var}(2X)$ with the variance of $X_1 + X_2$ for independent copies. [2 marks]

$\text{Var}(2X) = 4(4) = 16$.

Independent sum variance is $4 + 4 = 8$.

Answer: 16 versus 8.

5. Two normally distributed processing times come from the same machine cycle and are strongly related. Can you automatically add their variances? [2 marks]

Dependence may produce nonzero covariance; independence was not established.

Answer: No.

6. Independent normal X, Y have means 20,12 and variances 9,4. Find $P(X - Y < 5)$. [3 marks]

$X - Y \sim N(8, 13)$.

Standardise 5 using SD $\sqrt{13}$.

Answer: 0.2027.